MHD TURBULENT NATURAL CONVECTION IN A LIQUID METAL FILLED SQUARE ENCLOSURE

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Numerical two-dimensional study is carried out on the turbulent natural convection of molten sodium (Pr = 0.01085) in a square enclosure heated from one vertical wall and cooled from an opposing vertical wall. A magnetic field was applied in direction that its perpendicular, parallel and inclined with gravity vector. Turbulent natural convection occur at $Ra = 10^{10}$ and magnetic field strength change from $Ha = 0$ to 44970 with magnetic field orientation angle $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and $90^\circ$. The (k-$\varepsilon$) turbulence model was used. When a magnetic field is tilted in any angle, but not perpendicular or parallel with the gravity vector, the electromagnetic force is weaken and it’s least effect on flow pattern and temperature distribution from perpendicular direction. The magnetic field in any direction decreases the Nusselt number, but the magnetic field perpendicular with the gravity vector has more effect from than the other direction.

Keywords: MHD, Turbulence, Natural Convection

1. INTRODUCTION

The effect of the magnetic field has important application in engineering such as magnetic cooling, magnetic refrigerator, water treatment device, corrosion inhibition treatment, magnetohydrodynamics (MHD) power generation, plasma techniques and crystal growth (i.e. Czochralski system).

In plasma techniques the liquid metal is used as a coolant for fusion reactor blanket, and due to liquid metal susceptibility to the magnetic field that is present in the blanket, its flow pattern and heat transfer rate are influenced. In manufacturing of compounds semiconductors (i.e. GaAs) the special manufacturing process is used like Czochralski method, in this method the external magnetic field is applied to the melted of metal to suppress convective transport of depant in the melt and growing high-quality crystals.

In the last two problems, the external magnetic field effect on convection heat transfer by slow or inhibit the motion of liquid and this cause to decrease the convection heat transfer rate. At certain value of a magnetic field strength the turbulent flow change to laminar and Further increases in magnetic field strength the flow is quiescent, then heat transfer change from convection to conduction.

Many experimental, analytical and numerical studies have been made on the natural convection of electrically conducted fluids in presence of magnetic field[1-13]. Most of the previous studies apply magnetic field in perpendicular or parallel direction with the gravity vectors, no more existing studies apply magnetic field in the direction inclined with gravity vector. Also the lack is found in study numerically the effect of a magnetic field on a turbulent natural convection, by using a (k-$\varepsilon$) model for turbulence.
When the direction of a magnetic field is perpendicular to the gravity vector, the flow induced by the buoyant force crosses it. In that case, a term for the electromagnetic force appears in the momentum equation for the vertical velocity component. A term for the buoyancy force also appears in this equation. Therefore, the boundary layer approximation is applicable, so the equation is simplified as in [1,2]. But, when the direction of the magnetic field is parallel to the gravity vector, in this case, a term for the electromagnetic force appears in the momentum equation for the horizontal velocity component and the buoyancy force appear in the momentum equation for the vertical velocity component. The momentum equations for the velocities both parallel and perpendicular to the gravity must be solved as in [5].

When the direction of the magnetic field is neither perpendicular nor parallel to the gravity vector, but it is tilted in angle with the gravity vector (Fig. 1), the magnetic field interacts with the velocity components that are parallel and perpendicular to the gravity vector. And, a term for the electromagnetic force appears in the momentum equations for the vertical and horizontal velocity components and the buoyancy force appear in the momentum equation for the vertical velocity component. The momentum equations for the velocities both parallel and perpendicular to the gravity must be solved as in [6].

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In the present work, study is made on the numerical two-dimensional turbulent (Ra=10^{10}) natural convection of liquid metal (molten sodium) fill the square enclosure with differently heated vertical side walls, and imposed to external homogeneous magnetic field inclined in angle with gravity vector. Then compare the effect of inclination for the magnetic field with the perpendicular and parallel direction.

2. PROBLEM FORMULATION

A schematic of the physical situation to be investigated is shown in Fig. 1. The vertical walls of height H located at x = 0 and x = L are isothermal at different temperatures of T_h and T_c (T_h - T_c = 25 °C) respectively. The horizontal walls of width L are insulated at y = 0 and y = H, and all four walls are electrically and isothermally insulated boundaries. The flow in a square enclosure is subjected to uniform external magnetic field. The fluid within is assumed to have constant properties except insofar as the buoyancy is concerned, i.e. the Boussinesq approximation of linear temperature dependence of density is utilized. The flow is treated as steady, depending on the Rayleigh number and Hartmann number. The Rayleigh number chosen for turbulent flow is 10^{10}, and is subjected to different values of Hartmann number at different orientations of the external magnetic field (θ = 0°, 30°, 45°, 60° and 90°). The external magnetic field intensity for turbulent flow range between 0.0005 to 0.4 Tesla.

The liquid metal move in boundary layer (a region adjacent to the solid wall) is faster than in core region of the enclosure, it tends to pull out the field lines in the flow direction, so that the field acquires a component magnetic in the direction of flow. This feature is neglected in our research and assumed that the fluid motion does not affect the imposed magnetic field.

The no-slip condition is applied on the velocity at all four walls, and friction is calculated by invoking ‘wall function’.

3. CONSERVATION EQUATIONS

The fluid in the enclosure received both the buoyancy forces resulting from heat transfer through side walls and the electromagnetic force resulting from convection of fluid in an uniform magnetic field. The flow in a square enclosure is two-dimensional, subject to a uniform magnetic field B_0 of a constant magnitude B. The orientation of the magnetic field forms an angle θ with the horizontal axis.

By using Ohm’s law without Hall Effect and electrically insulated boundaries, the electric current density is

\[ J = \sigma (V \times B) \]  \hspace{1cm} (1)

the electromagnetic force

\[ F_{EM} = J \times B \]  \hspace{1cm} (2)

The ensemble average of the fluctuation electromagnetic force in the momentum equation was neglected. The induced magnetic field is small compared to the applied magnetic field (Rm << 1), so

\[ B = B_o \]  \hspace{1cm} (3)

and

\[ B = B_x \hat{i} + B_y \hat{j} \]  \hspace{1cm} (4)
the orientation angle of a magnetic field

\[ \theta = \arctan \frac{B_y}{B_x} \]  

(5)

The turbulent flow is described by the continuity, momentum and energy equations as follow:

Continuity equation

\[ \frac{\partial}{\partial x} \rho u + \frac{\partial}{\partial y} \rho v = 0 \]  

(6)

x-direction momentum equation

\[ \frac{\partial}{\partial x} \rho u \frac{\partial u}{\partial x} + \frac{\partial}{\partial y} \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial x} \left( \mu_{\text{eff}} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_{\text{eff}} \frac{\partial v}{\partial y} \right) \]

\[ (\mu_{\text{eff}} \frac{\partial v}{\partial y}) + S_u \]  

(7)

y-direction momentum equation

\[ \frac{\partial}{\partial x} \rho u \frac{\partial v}{\partial x} + \frac{\partial}{\partial y} \rho v \frac{\partial v}{\partial y} = \frac{\partial}{\partial x} \left( \mu_{\text{eff}} \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_{\text{eff}} \frac{\partial v}{\partial y} \right) \]

\[ (\mu_{\text{eff}} \frac{\partial v}{\partial y}) + S_v \]  

(8)

Thermal energy equation

\[ \frac{\partial}{\partial x} \rho u T + \frac{\partial}{\partial y} \rho v T = \frac{\partial}{\partial x} \left( \Gamma_{\text{eff}} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_{\text{eff}} \frac{\partial T}{\partial y} \right) + S_T \]

(9)

The value of \( \mu_{\text{eff}} \) in equations (7) and (8) is

\[ \mu_{\text{eff}} = \mu + \mu_t \]  

(10)

and

\[ S_u = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left( \mu_{\text{eff}} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_{\text{eff}} \frac{\partial v}{\partial y} \right) + \sigma B^2 (v \cos \theta \sin \theta - u \sin^2 \theta) \]  

(11)

\[ S_v = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left( \mu_{\text{eff}} \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_{\text{eff}} \frac{\partial v}{\partial y} \right) + \rho g \beta (T - T_e) + \sigma B^2 (u \cos \theta \sin \theta - v \cos^2 \theta) \]

\[ \Gamma_{\text{eff}} = \frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \]  

(13)

In thermal energy equation the radiation heat transfer, the viscous dissipation, pressure work and Joule heating are ignored, so the source term \( S_T \) becomes

\[ S_T = 0.0 \]  

(14)

It is known that in the presence of a sufficiently strong magnetic field the turbulent fluctuations become anisotropic, which implies important consequences for the properties of the turbulence and possibly requires modification of numerical models. Specific manifestation of the anisotropy may vary but the principle mechanism is always the elongation of flow structures (turbulent eddies) along the lines of the magnetic field.

From the previous situation the appropriate method to find the value of turbulent dynamic viscosity \( \mu_t \) is by use the DNS or LES method, e.g. explicitly resolve all the scales of the flow without using any turbulence model. These methods require the solution region made fine enough to encompass the smallest eddy scale (eddy Reynolds number is unity) and at the same time large enough to include the largest eddies. But this need high sufficient number of grid points and it’s not available on personal computer CPU 1.7 A.

Another method can be follow to find value of \( \mu_t \) is by use the k-\( \varepsilon \) model of turbulence, this model must be modify due to magnetic field effect\[14\].

The empirical constants values for electromagnetic dissipation terms, which is effect in k- and \( \varepsilon \)-equation are vary between 0.0 to 2.0 and are decreases as the magnetic field increase. Also according to Smolentsev et al.\[15\] the anisotropy in the turbulence structure associated with the Hartmann effect has not been introduced. The turbulent kinetic energy equation, k, and the dissipation rate of turbulent kinetic energy equation, \( \varepsilon \), has not modified due to the combination of electromagnetic and buoyancy force effect.

The turbulent flow assumed to be characterized by its kinetic energy k and dissipation rate \( \varepsilon \) thus; \( \mu_t \) is determined by the local value of \( \rho \), k, \( \varepsilon \) and for dimensional homogeneity

\[ \mu_t = C_k \rho \frac{k^2}{\varepsilon} \]  

(15)

The turbulence parameters k and \( \varepsilon \) are not modified due to the magnetic field effect and are given by Dubke et al.\[16\], as the following form;

Turbulence energy, k

\[ \frac{\partial}{\partial x} \rho u k + \frac{\partial}{\partial y} \rho v k = \frac{\partial}{\partial x} (\Gamma_k \frac{\partial k}{\partial x}) + \frac{\partial}{\partial y} (\Gamma_k \frac{\partial k}{\partial y}) + G - \rho \varepsilon \]  

(16)

Dissipation rate, \( \varepsilon \)

\[ \frac{\partial}{\partial x} \rho u \varepsilon + \frac{\partial}{\partial y} \rho v \varepsilon = \frac{\partial}{\partial x} (\Gamma_\varepsilon \frac{\partial \varepsilon}{\partial x}) + \frac{\partial}{\partial y} (\Gamma_\varepsilon \frac{\partial \varepsilon}{\partial y}) + C_1 \frac{\varepsilon}{k} G - C_2 \frac{\rho \varepsilon^2}{k} \]  

(17)

where;

\[ \Gamma_k = \frac{\mu_{\text{eff}}}{Pr_k} \]  

(18)
\[ \Gamma_c = \frac{\mu_{eff}}{Pr_c} \quad (19) \]

\[ G = \mu_t \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \]  

(20)

The turbulence model constants \( C_1, C_2, C_3 \) and \( C_D \) used in calculation are given in Table 1.

<table>
<thead>
<tr>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.44</td>
<td>1.92</td>
<td>0.09</td>
<td>1</td>
</tr>
</tbody>
</table>

The term for kinetic energy generation by buoyancy in equations (16) and (17) is ignored, this term according to Mason and Seban\(^\text{[17]}\), produces a buoyancy in equations (16) and (17) is ignored, this term according to Mason and Seban\(^\text{[17]}\), produces a slight increase in the turbulence level but has a negligible effect upon the heat-transfer coefficient.

According to Markatos\(^\text{[18]}\), the turbulent Prandtl number \( Pr_t = 1 \) and the value of the Prandtl number for \( k \) (Prk) is taken as 0.9 and for \( \varepsilon \) (Pr\( \varepsilon \)) is derived from the relation; \( \kappa^2/C_1 \)\(^2/(C_2-C_1) \) where \( \kappa \) is the Von Karman constant (\( \kappa = 0.42 \)).

4. METHOD OF SOLUTION

The SIMPLE algorithm by Patankar\(^\text{[19]}\) was applied to solve the conservation equation of mass, momentum and energy.

The transport equations for continuity equation (6), momentum equation (7,8), energy equation (9), turbulence energy equation (16) and dissipation rate equation (17) all have the general form in two-dimensional geometry:

\[ \frac{\partial}{\partial x} (\rho u \Phi) + \frac{\partial}{\partial y} (\rho v \Phi) = \frac{\partial}{\partial x} \left( \Gamma_\phi \frac{\partial \Phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_\phi \frac{\partial \Phi}{\partial y} \right) + S_\phi \]  

(21)

Equation (21) is re-arrangement of algebraic equation of the form

\[ (\sum_i a_{i} - S_p) \Phi_p = \sum_i a_i \Phi_i + S_a \]  

(22)

The set of finite-difference equations is solved by using an iteration method, starting from the solution of a previous run as initial condition. Numerical stability is enhanced by the choice of approximate relaxation factor \( f \) defined as

\[ \Phi_p^{n} = f \Phi_p + \left( 1 - f \right) \Phi_p^{n} \]  

(23)

where, \( \Phi_p^{n} \) is the value of \( \Phi \) from the previous iteration, \( \Phi_p \) is the values obtained from the solved equation (22) and \( \Phi_p^{n} \) is the new value. The value of the relaxation factor \( f \) can be in the range of 0 < \( f \) ≤ 1.

The nonuniform mesh of (x,y) = 160 \times 60 with \( (\Delta x)_{\min} = 0.0007 \) m and \( (\Delta y)_{\min} = 0.0018 \) m has been used for the location of the first grid point near the walls.

5. RESULTS AND DISCUSSION

The solution was computed for Prandtl number equal to 0.01085 and Rayleigh number equal to \( 10^{10} \).

5.1 The Stream Functions and Isotherms

For turbulent flow, at \( Ra = 10^{10} \) and without magnetic field effect (Ha = 0) the stream functions and isotherms are illustrated in Figures 2a&b, respectively. And with magnetic field effect, the stream functions are illustrated in Figures 3 and 4 and isotherms are illustrated in Figure 5 for different external magnetic field orientation angle. From these Figures one can observe the change in stream functions shape and value, in general, when increases the value of Hartmann number the stream function value decreases.

The effect of a magnetic field orientation angle (\( \theta \)) are shown clearly as in Fig. 3. When \( \theta = 0^\circ \) the magnetic field lines are in x-direction and the electromagnetic force effect in y-momentum equation only, the stream function change it’s shapes to approximate elliptic shape as illustrated in Fig. 3a.

But at \( \theta = 90^\circ \) the magnetic field lines are in y-direction and the electromagnetic force effect in x-momentum equation only, the stream function takes the shape as shown in Fig. 3e and two vortices appear, each one locate close to the enclosure heated and cooled wall. At \( \theta = 90^\circ \) the values of stream function is greater than that at \( \theta = 0^\circ \), this is due to the fact that the effect of a magnetic field parallel to the gravity vector is less than that for a field normal to the gravity vector\(^\text{(i)}\).

If the magnetic field lines are inclined with \( 0^\circ < \theta < 90^\circ \) the magnetic field lines analyze to two components, x- and y-components which are represented by \( B_x \) and \( B_y \) respectively, and the electromagnetic force effect in both y-and x-momentum equations. To illustrate this, assume the magnetic field lines are inclined with \( \theta = 30^\circ \) the stream functions change its shape as shown in Fig. 3b, and its value increases slightly from state at \( \theta = 0^\circ \). This means, that when the inclined the magnetic field lines from the perpendicular state (x-direction) the magnetic field strength will weaken and this would cause to increase the stream function value due to increases in values of velocity components. Now if \( \theta = 45^\circ \) the stream functions take the shape shown in Fig. 3c and the central stream function are more elongated from that in state at \( \theta = 30^\circ \), also the stream function value have more increase. At \( \theta = 60^\circ \) the two vortices appear as illustrated in Fig. 3c, these vortices appear also at \( \theta = 90^\circ \).
These changes in flow pattern are dependent on the value of the electromagnetic force effect in x- and y-momentum and which one is greater than the other. Mathematically, when external magnetic field applied at $\theta = 0^\circ$ or $90^\circ$ the electromagnetic force work as sink only and have great effect on flow pattern. But when external magnetic field applied at $0^\circ < \theta < 90^\circ$ ($\theta = 30^\circ$, $45^\circ$ and $60^\circ$) the electromagnetic force work as a sink and source so its effect is weakened on flow pattern.

The temperature distribution is also affected by magnetic field orientation angle $\theta$ as illustrated by Fig. 5, the change in isotherms is cleared between $\theta = 0^\circ$ and $\theta = 90^\circ$ as shown in Fig. 5a & e. The isotherms are denser on the lower part of the hot vertical wall and on the top part of the cold vertical wall. This indicates that heat transfer through these parts of the walls is greater from the other parts of the vertical wall. The isotherms in Fig. 5 are different from that in Fig. 2b, this due to the effect of a magnetic field, and at very high value for Hartmann number the isotherms are changing from convection to conduction heat transfer profile.

5.2 The Average Nusselt Number

The average Nusselt number values of natural convection for liquid metal (molten sodium) in an enclosed enclosure have been calculated by Jalil et al. [20] at different values of Rayleigh number and without external magnetic effect ($Ha = 0$) and are presented in Table 2.

In turbulent flow at $Ra = 10^{10}$, the value of Nusselt number decrease with increases in the Hartmann number value at fixed orientation angle (i.e. $\theta = 0^\circ$, or any value) to reach to the value of Nusselt number at approximately $10^3 < Ra < 10^4$ as presented in Table 3. For example, in Table 2 when $Ra = 10^{10}$ and $Ha = 0$ the value of $Nu = 100.6$, but at $Ha = 5622$ and $\theta = 45^\circ$ the value of $Nu = 30.59$, this value of $Nu$ is closed to value of $Nu$ at $Ra = 10^8$ (see Table 2) and the flow is turbulent. When $Ha$ increases to 22490 at same orientation angle the value of $Nu$ is 5.057, this value is close to $Nu$ at $Ra = 10^5$ and the flow becomes laminar. Generally, when the value of $Ha$ increases more, the value of $Nu$ decreases more.

Table 3 shown that the value of Nusselt number at $\theta = 0^\circ$ is lesser than at $\theta = 90^\circ$ for all values of Hartmann numbers. Except that at very low Hartmann number ($Ha = 56.22$) the value of Nusselt number is leveling off at any angle $\theta$, and at very high Hartmann number ($Ha = 44970$) the value of Nusselt number at $\theta = 0^\circ$ is become greater than at $\theta = 90^\circ$.

At fixed value of Hartmann number, when there are increases in the value of external magnetic field orientation angle ($\theta > 0^\circ$), the value of Nusselt number increase to approach its value at $\theta = 90^\circ$. This means that for fixed value of $\theta$, the value of Nusselt number is depending on which value of electromagnetic force
Figure 3. Stream functions (mm²/sec) at Ra = 10^{10} and Ha = 5622; (a) θ = 0°; (b) θ = 30°; (c) θ = 45°; (d) θ = 60°; (e) θ = 90°.
Figure 4. Stream functions (mm²/sec) at Ra = 10^{10} and Ha = 44970; (a) θ = 0°; (b) θ = 30°; (c) θ = 45°; (d) θ = 60°; (e)
Figure 5. Isotherms (°C) at Ra = 10^{10} and Ha = 22490; (a) \( \theta = 0^\circ \); (b) \( \theta = 30^\circ \); (c) \( \theta = 45^\circ \); (d) \( \theta = 60^\circ \); (e) \( \theta = 90^\circ \)
is dominant in x- or y-momentum equation. For example, at $\text{Ha} = 2811$, the value of Nusselt number at $\theta = 30^\circ$ is less than at $\theta = 60^\circ$ because the electromagnetic force in y-direction has in dominant than in x-direction, and it has great effect on Nusselt number compared with that in x-direction.

The Table 3 is illustrated in Fig. 6. The main observations discussion so far can also be infer from this Figure, i.e. the relation between Nusselt number value and external magnetic field orientation angle at certain Hartmann number value.

In general, the range of Nusselt number value for $0^\circ < \theta < 90^\circ$ is lesser than the value of Nusselt number at $\theta = 0^\circ$ (Bx effect only) and greater than the value of Nusselt number at $\theta = 90^\circ$ (By effect only).

Table 3. Effect of Hartmann number on average Nusselt number at turbulent flow ($Ra = 10^{10}$)

<table>
<thead>
<tr>
<th>Ha</th>
<th>$0^\circ$</th>
<th>$30^\circ$</th>
<th>$45^\circ$</th>
<th>$60^\circ$</th>
<th>$90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>56.22</td>
<td>100.4</td>
<td>100.4</td>
<td>100.4</td>
<td>100.4</td>
<td>100.4</td>
</tr>
<tr>
<td>84.32</td>
<td>100.3</td>
<td>100.4</td>
<td>100.4</td>
<td>100.4</td>
<td>100.4</td>
</tr>
<tr>
<td>562.2</td>
<td>95.29</td>
<td>95.29</td>
<td>97.66</td>
<td>98.81</td>
<td>99.93</td>
</tr>
<tr>
<td>2811</td>
<td>44.59</td>
<td>48.86</td>
<td>55.40</td>
<td>65.96</td>
<td>83.83</td>
</tr>
<tr>
<td>5622</td>
<td>23.33</td>
<td>26.04</td>
<td>30.59</td>
<td>39.04</td>
<td>60.10</td>
</tr>
<tr>
<td>11240</td>
<td>10.61</td>
<td>11.41</td>
<td>13.19</td>
<td>16.43</td>
<td>24.23</td>
</tr>
<tr>
<td>22490</td>
<td>4.285</td>
<td>4.615</td>
<td>5.057</td>
<td>5.090</td>
<td>5.156</td>
</tr>
<tr>
<td>44970</td>
<td>2.307</td>
<td>2.414</td>
<td>2.552</td>
<td>2.410</td>
<td>2.272</td>
</tr>
</tbody>
</table>

6. CONVERGENCE

residual value from the sum of the equation (22) for $u$, $v$ and $p$ values. The residual value must be equal to zero, but for turbulent flows in the square enclosure without effect of magnetic field (only gravitation natural convection effect in momentum equation) the residual value increases to value $10^{-3}$, and with the effect of magnetic field the residual value increases to $5 \times 10^{-3}$.

The problems are solved by personal computer CPU 1.7A, in turbulence flow $Ra = 10^{10}$ this computer need more run-times for computing and is approximately 180 min without effect of external magnetic field. When a low strength of magnetic field is applied the run-time decrease, and when increasing the strength, the run-times decrease more, but at very high strength of magnetic field the run-times is increases.

To economize and decrease on run-times each case for effect of external magnetic field starts from case without magnetic field effect ($Ha = 0$). The value of under relaxation factor ($\alpha$) choice for $u$ and $v$ equations for turbulent flow without effect of magnetic field is $0.05$, when magnetic field effect was applied this value decreases to $0.005$.

7. CONCLUSION

The two-dimensional numerical model was applied to steady the effect of the direction of external magnetic field applied on liquid metal (molten sodium) fills a square enclosure heated from one vertical side wall and cooled from an opposing wall is presented as well as the boundary conditions for all variables. The direction of the external magnetic field change according to $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and $90^\circ$ in turbulent natural convection heat transfer for $Ra = 10^{10}$ and $Ha = 0 - 44970$. In general, a magnetic field in $x$-direction is more effective on flow pattern and temperature distribution than from $y$-direction and the orientation effect of magnetic field depends on which magnetic field component $B_x$ or $B_y$ is in demand.

The value of Nusselt number decreases with increase in Hartmann number value at same orientation of magnetic field, and the change in Nusselt number value with magnetic field orientation dependent on which value of electromagnetic force is in demand in $x$-direction or $y$-direction.

The study can be developed to three-dimensional and can deal unsteady state or non-uniform magnetic field.
*NOMENCLATURE*

- B: magnetic induction (Tesla)
- B_x: magnetic induction in x-direction (Tesla)
- B_y: magnetic induction in y-direction (Tesla)
- B_e: external magnetic field (Tesla)
- F_{EM}: electromagnetic force (N/m²)
- G: kinetic energy generation by shear (Joule)
- H: Hartmann number, \((\sigma H)/\nu\)
- Ha: Hartmann number, \((\sigma H)/\nu\)
- L: enclosure width (m)
- M: magnetic induction (Tesla)
- p: pressure (N/m²)
- Pr: Prandtl number, \(\nu/\alpha\)
- Ra: Rayleigh number, \(g/(\beta L^3)/L^3\)
- Rm: magnetic Reynolds number
- T: temperature (K)
- T_c: cold wall temperature (K)
- T_h: hot wall temperature (K)
- u: velocity of x-component (m/sec)
- v: velocity of y-component (m/sec)
- w: velocity of z-component (m/sec)
- x: coordinate (m)
- y: coordinate (m)

*Greek Letters*

- \(\theta\): external magnetic field orientation angle (degree)
- \(\beta\): volumetric coefficient of expansion (K⁻¹)
- \(\mu\): dynamic viscosity of fluid (kg/m sec)
- \(\rho\): density of fluid (kg/m³)
- \(\sigma_e\): electric specific conductivity of fluid (S/m)
- \(\Gamma\): diffusion coefficient (kg/m sec)
- \(\varepsilon\): dissipation energy rate (m²/sec³)

*Subscripts*

- eff: effective
- t: turbulence

*REFERENCES*