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KINEMATIC NOMOGRAPHS OF EPICYCLIC-TYPE TRANSMISSION MECHANISMS

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يقدّم هذا البحث طريقه جديده لحساب النسب السرعيه لآليات نقل الحركه ذات التروس الكواكبيه. تم تمثيل النظام بالاعتماد على النظريه الرصينه لتمثيل الانظمه بالمخططات، ومنها أمكن الاهتداء الى الكيانات البنائيه الترسيه الاساسيه. وقد قاد التعرف على هذه الكيانات البنائيه الى أنشاء المخططات النوموغرافيه آليا" بطريقه منهجيه منظمه. يتحرى هذا البحث ، بأستخدام المخططات النوموغرافيه، السمات الكينماتيكيه المقترنه بمختلف الكيانات البنائيه الترسيه الاساسيه، ومنها أمكن التعبير عن النسبه السرعيه الكليه لآليه ترسيه كوكبيه بدلالة نسبة أسنان ازواج التروس المكونه لها. أن الميزه الاساسيه للمخططات النوموغرافيه هي بساطتها. من مخطط نوموغرافي واحد, يمكن أن نبصر السرع الدورانيه لكل الاجزاء الدوّاره. و يمكن الحصول على التعابير ألرياضيه لكل النسب السرعيه لكيان بنائي ترسي و/ أو لآلية ترسيه كوكبيه بالمشاهد ه فقط دون ألحاجه إلى حل المعادلات مرارا" وتكرارا" ودون تحديد قياس كل ترس ابتداء". بالاضافه إلى ذلك ، فأنها تعطي رؤيا أفضل لتأثير الكيانات البنائيه ألترسيه الاساسيه ومقاسات التروس على ألنسبه السرعيه الكليه لآلية التروس الكواكبيه ، وهذا مفيد جدا" في التعرف على تتابع تعشيق التروس أثناء ألمرحله ألتصميميه لآليات نقل الحركه ذات التروس الكواكبيه.

A new methodology for the velocity ratio analysis of epicyclic-type transmission mechanism is presented. The well established graph theory is used to represent the system and then to detect the fundamental geared entities (FGEs). The identification of FGEs leads to automated construction of the kinematic nomographs in a systematic manner. Using nomographs the kinematic characteristics associated with various FGEs are investigated. Then, it is shown that the overall velocity ratio of an epicyclic gear mechanism (EGM) can be expressed in terms of the gear ratios of its gear pairs. From a single nomograph one can visualize the angular velocities of all the links of an EGM. The main advantage of kinematic nomographs is its simplicity. Also, algebraic expressions for all of the velocity ratios of an FGE or/and an EGM can be easily obtained by observation without the need to solve equations repeatedly and without specifying the exact size of each gear. In addition, it provides a better insight of the effects of the FGEs and their gear sizes on the overall velocity ratio of an EGM. This is very helpful in the identification of a clutching sequence during the design phase of an epicyclic -type transmission mechanism.

1. INTRODUCTION

Most automatic transmission mechanisms employ epicyclic gear trains (EGTs) to achieve a set of desired speed ratios. Typically, the central axis of an EGT is supported by bearings housed in the casing of an automatic transmission. The EGT and the casing form a fractionated mechanism called an EGM. Figure 1 shows an EGM employing the Simpson gear train as the multi-speed reduction unit.

In an EGM, the velocity ratio is defined as the ratio of the input shaft velocity to the output shaft velocity. Various velocity ratios are obtained by using clutches to connect various links to the input power source and to the casing of a transmission gearbox, respectively. Typically, a rotating clutch is used for connecting two rotating links and a band clutch is used to fix a link to the casing. In Figure 1 rotating and band clutches are denoted by C and B, respectively. Also it is always possible to achieve a direct drive by locking all the links in the EGT together such that they rotate as a single link.

The velocity ratios selected for a transmission are tailored for vehicle performance. Typically, they include a first gear for starting, a second and a third gear for passing, and a fourth gear for fuel economy at road speeds. A table depicting a set of speed ratios and their clutching conditions is called a clutching sequence. Table 1 shows the clutching sequence of the transmission shown in Figure 1, where an X*i* indicates that the corresponding clutch is activated on the *i*th link for that gear. For example, when the mechanism is in the first gear, the rotating clutch C1 and the band clutch B1 are activated. Hence, link 4 is connected to the input power and link 1 is fixed to the casing.

Table 1. Clutching sequence of the mechanism shown in Figure 1

	Activated clutches			
Range	C1	C2	B1	B2
First	X4		X1	
Second	X4			X3
Third	X4	X3		
Reverse		X3	X1	

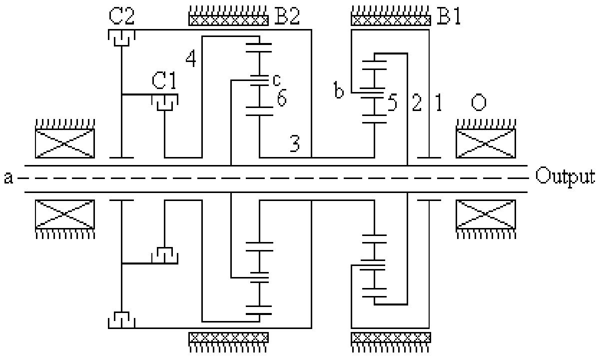


Figure 1. A Typical transmission mechanism.

2. LITERATURE REVIEW

For the kinematic analysis of EGMs, various approaches such as the relative velocity method[1-3], energy method[4], bond graph method[5] and the vector-loop method[6,7] have been proposed. Freudenstein and Yang[8] introduced the concept of fundamental circuit to analyze EGTs. The concept was further extended by other researchers[9-10]. The concept of fundamental circuit is a powerful tool for automated analysis of EGTs. However, the analysis involves the solution of a set of linear equations for all the kinematic variables. It does not provide much insight into mechanics of an EGT.

In addition, some other studies have concentrated on structural and dimensional syntheses[11-16]. The kinematic analysis for geared mechanisms has been nearly perfected, the challenge lies in proposing new simplified methods and subsequent clear visualization of the rotational velocities and torques acting on the EGMs. However, relatively little work has been done to find the overall velocity ratios of EGMs in a systematic manner. Notable exception is the work by Hsieh and Tsai[10,17], who applied the kinematic characteristics of FGE to the overall velocity ratio analysis. In their method, the overall velocity ratio of an EGM is expressed in terms of the velocity ratios of its FGEs by decomposing an EGM into two subsystems and each subsystem into two subsystems until the lowest level subsystem contains only FGEs. First of all, they introduce four operation modes of FGEs and use them as the building blocks of kinematic analysis. The velocity ratios associated with these modes are written in terms of gear size ratios. Second, the FGE diagram is used also to facilitate the velocity ratio analysis. Finally, they assign the input, output and fixed links to the EGM. The overall velocity ratio associated with each possible assignment of the input, fixed and output links of an EGM is written in terms of the velocity ratios of its FGEs. While this technique is valid for solving the transmission design problem, it limits the solution to a single velocity ratio each time, and it is a lengthy computational process, introducing many opportunities for error on the part of the designer. This inevitably reduces the efficiency of this solution technique. In this paper, a more efficient solution technique is developed to overcome this shortcoming. This paper applies nomographs of FGTs for the kinematic analysis of EGTs to achieve this goal.

3. NOMOGRAPHS

Nomograph is defined as three or more axes, or scales, arranged such that problems of three or more variables can be solved using a straightedge. In the particular case of EGTs, a nomograph can be constructed using three or more vertical parallel axes[18].

A basic EGT consists of a sun gear, a ring gear, a planet, and a carrier as shown in Figure 2. Figure 3 shows the basic form of the graph to be created for a basic EGT. The ωc axis is chosen to pass at the origin; also the ωp and ωc axes are chosen to be one unit apart.

The term "gear ratio" is used in this paper to denote the ratio of a meshing gear pair, while the term "velocity ratio" is used to denote the velocity ratio between the input link and the output link of an EGM. $N_{p,x}$ is the gear ratio defined by a planet gear p with respect to a sun or ring gear x

$$N_{p,x} = \pm Zp / Zx \tag{1}$$

where Zp and Zx denote the numbers of teeth on the planet and the sun or ring gear, respectively, and the positive or negative signs depends on whether x is a ring or sun gear. Considering the kinematics of a fundamental circuit, the fundamental circuit equation can be written as Buchsbaum and Freudenstein[11]:

$$(\omega_x - \omega_c)/(\omega_p - \omega_c) = N_{p,x} \tag{2}$$

Equation (2) can be re-written for the ring and sun gears as follows

$$(\omega_r - \omega_c)/(\omega_p - \omega_c) = N_{p,r} \tag{3}$$

$$(\omega_s - \omega_c)/(\omega_p - \omega_c) = N_{p,s} \tag{4}$$

Dividing Eq. (4) by (3) yields a kinematic equation relating the angular velocities of the two gears r and s, and the carrier c as the three ports of communication with the external environment.

Let the symbol $R^{c}_{s,r}$ denote the velocity ratio between links s and r with reference to link c. That is

$$R^{c}_{s,r} = (\omega_s - \omega_c)/(\omega_r - \omega_c) = N_{p,s} / N_{p,r} \tag{5}$$

But from the nomograph shown in Figure 3, we can find directly that

$$R^{c}_{s,r} = N_{p,s} / N_{p,r} \tag{6}$$

$$R^{c}_{r,s} = N_{p,r} / N_{p,s} = 1/ R^{c}_{s,r} \tag{7}$$

Also from the same nomograph

$$R^{r}_{s,c} = (N_{p,s} - N_{p,r}) /(-N_{p,r}) = 1 - (N_{p,s} /N_{p,r}) = 1-R^{c}_{s,r} \tag{8}$$

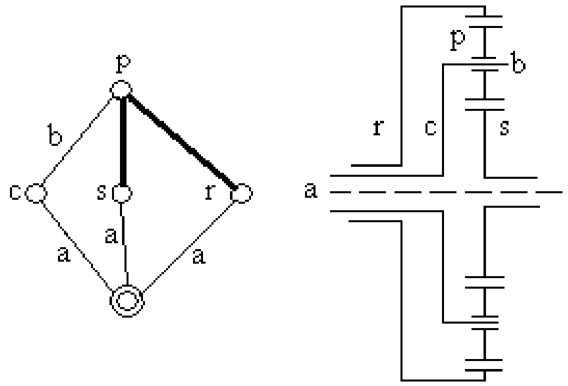


Figure 2. A basic epicyclic gear train.

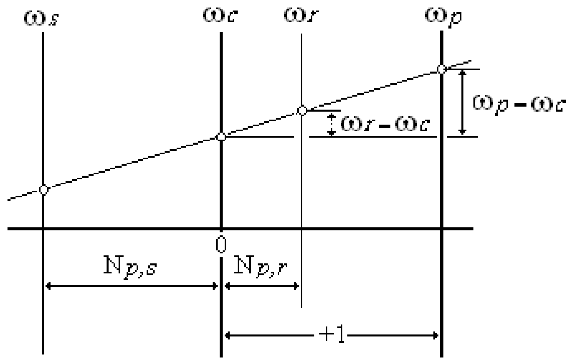


Figure 3. Nomograph for the basic epicyclic gear train.

$$R^{s}_{c,r} = 1/R^{s}_{r,c} = 1/(1-R^{c}_{r,s}) = 1/(1-(1/R^{c}_{s,r})) = R^{c}_{s,r} / (R^{c}_{s,r}-1) \quad (9)$$

After some examination to the nomograph shown in Figure 3, one find that from a single nomograph one can visualize the angular velocities of the three gears *r,s* and *p*, and the carrier *c*. Also algebraic expressions for all of the velocity ratios of a basic EGT can easily be obtained by observation without the need to solve equations repeatedly and without first selecting of the exact size of each gear. Also, it provides a better insight of the effects of gear sizes on the overall velocity ratio.

One velocity ratio can be used to determine the remaining five as shown in Table 2.

Table 2. Velocity Ratio Relations

Velocity Ratios	$R^{c}_{s,r}$	$R^{c}_{r,s}$	$R^{r}_{s,c}$	$R^{r}_{c,s}$	$R^{s}_{r,c}$	$R^{s}_{c,r}$
Velocity Ratio Relations	e	1/e	1-e	1/(1-e)	(e-1)/e	e/(e-1)

4. CANONICAL GRAPH REPRESENTATION

Buchsbaum and Freudenstein[11] used the graph to represent an EGT. In a graph representation, links are represented by vertices and joints are represented by edges. The gear joints are represented by thick edges, revolute joints are represented by thin edges, and the thin edges are labeled according to their axis locations in space. Figure 4 shows the graph representation of the transmission mechanism shown in Figure 1.

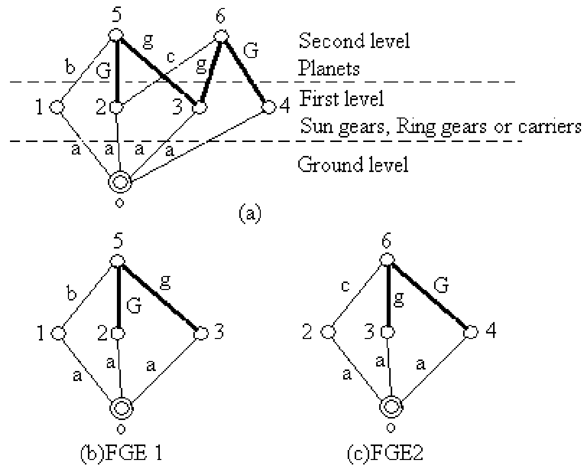


Figure 4. Canonical graph and FGEs of the mechanism in Figure 1

When there are three or more coaxial links in a mechanism, the joints among these coaxial links can be rearranged without affecting the functionality of the mechanism. This may result in the problem of isomorphism. Two graphs are said to be isomorphic if there exists a one-to-one correspondence between their vertices and edges which preserve the incidence and labeling. For EGTs, two mathematically non-isomorphic graphs can represent mechanisms that are kinematically equivalent. Such graphs are called pseudo-isomorphic graphs[13].

In order to avoid the problem of pseudo-isomorphic due to the existence of coaxial links, a canonical graph representation was suggested[15]. In a canonical graph representation, the joints among some coaxial links are rearranged such that all the thin-edged paths originated from the root (fixed link) have distinct edge labels. Furthermore, the vertices can be divided into several levels. The ground-level vertex, called the root, denotes the casing. The first-level vertices denote the coaxial sun gears, ring gears, and carriers. The second- level vertices denote the planets. The canonical graph representation of the EGM shown in Figure 1 is sketched in Figure 4. In Figure 4(a), link o is the ground-level vertex, links 1 through 4 are the first-level vertices, and links 5 and 6 are the second-level vertices. Label denotes the common axis of the turning pair joints which join the first-level vertices and the ground-level vertex. Labels b and c denote the axes of the turning pair joints which join the second-level vertices and the first-level vertices. Labels G and g denote the internal and external gear joints, respectively. A literature survey reveals that all existing automatic transmission mechanisms have their links distributed up to the second level[19,20]. In this paper, only those EGMs with their links distributed up to the second level will be considered.

4.1 Fundamental Geared Entity

Due to the fact that an EGM can be decomposed into several FGTs[16], the kinematics of an EGM are closely related to the kinematics of each individual FGE.

Chatterjee and Tsai[15] defined the FGE as a mechanism represented by a subgraph formed by a single second-level vertex or a chain of heavy-edge connected second-level vertices together with all the lower vertices connecting them to the root. Figures 4(b) and 4(c) show the graphs of two FGEs identified from Figure 4(a). The FGEs can be categorized into single-planet FGE, double-planet FGE and triple-planet FGE as shown in Figures 5, 7 and 10.

4.2 Kinematic of a Single-Planet FGE

In a single-planet FGE, the coaxial links include a carrier and several first-level gears meshing with one planet gear. Figure 5 shows a typical single-planet FGE Figure 6 shows a nomograph for this FGE.

For an FGE or an EGM with m coaxial links, there are (m)(m-1)(m-2)/6 possible sets of three coaxial links. Given three coaxial links *i*, *j*, and *k*, there are 3!=6 possible velocity ratios between these three coaxial links. A total of 3!(m)(m-1)(m-2)/6 velocity ratios between the m coaxial links .

The velocity relationship among these coaxial links should be established in order to effectively compute the velocity ratios of a transmission mechanism. This can be best accomplished by using a single nomograph from which all the angular velocities can be visualized. Also, algebraic expressions for all of the velocity ratios can easily be obtained by observation.

4.3 Kinematic of a Double-Planet FGE

In a double-planet FGE, there are two meshing planets, one carrier, and several coaxial gears meshing with either one of the two planets. Figure 7 shows a typical double-planet FGE. Figure 8 shows a nomograph for this FGE in terms of the planet gear *p1*. Figure 9 shows a nomograph for the same FGE in terms of the planet gear *p2*.

The gear ratio for the coaxial gears that are not meshing directly with the planet gear on which the nomograph is drawn and are meshing with the other planet gear can be found in terms of the gear ratio of the two planets ($N_{p1,p2}$) or ($N_{p2,p1}$) as

$$N_{p1,x} = N_{p1,p2} \, . \, N_{p2,x} \quad (10)$$

$$or \;\; N_{p2,x} = N_{p2,p1} \, . \, N_{p1,x} \quad (11)$$

$$where \;\; N_{p2,p1} = 1 / N_{p1,p2} \quad (12)$$

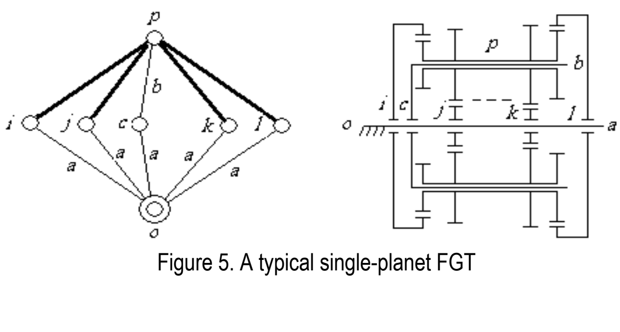


Figure 5. A typical single-planet FGT

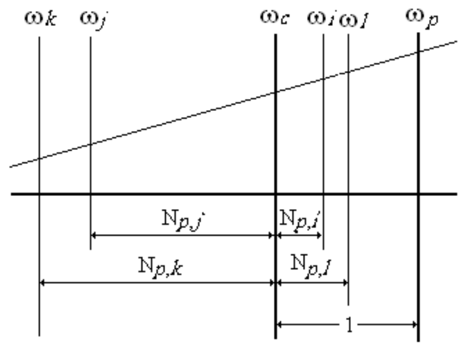


Figure 6. Nomograph for the single-planet FGT shown in Figure 5

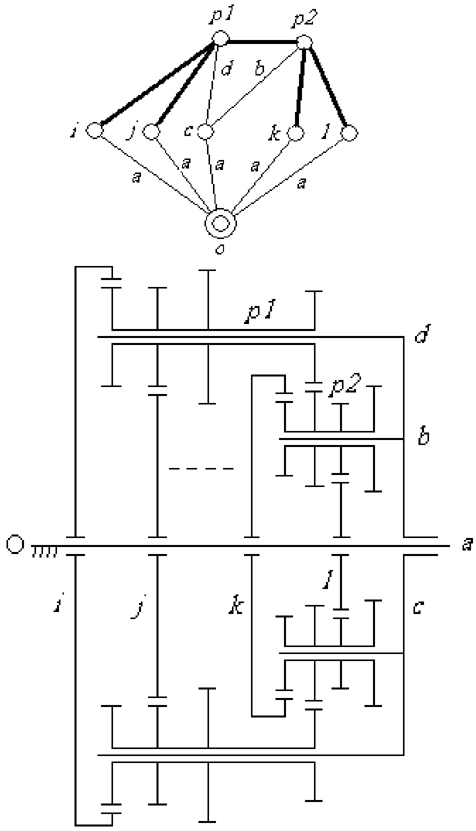


Figure 7. A typical double-planet FGT

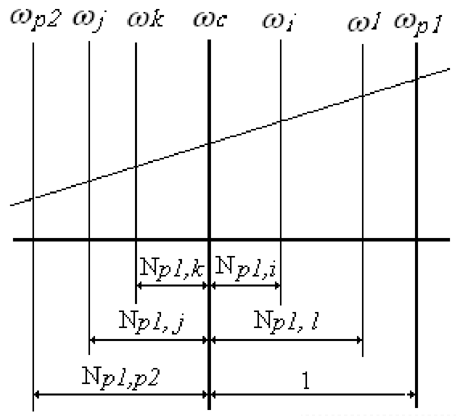


Figure 8. Nomograph for the double-planet FGT shown in Figure 7, in terms of the planet gear *p1*

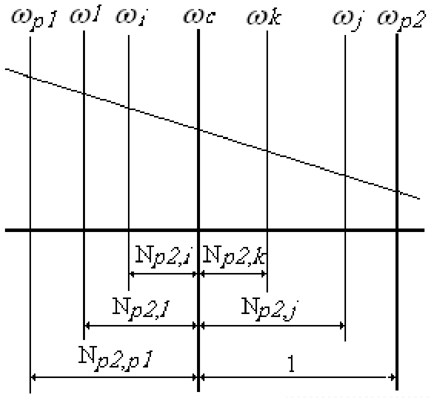


Figure 9. Nomograph for the double-planet FGT shown in Figure 7, in terms of the planet gear *p*2.

Again, algebraic expressions for all of the velocity ratios of this FGE can easily be obtained by observation, from Figure 8 or 9.

$$R^{c}_{j,k} = Np1,j \;/\; Np1,k \tag{13}$$

From equation (10)

$$Np1,k = Np1,p2 \,.\, Np2,k \tag{14}$$

$$\text{So } R^{c}_{j,k} = Np1,j \,/\, Np1,p2 \,.\, Np2,k \tag{15}$$

$$\text{and } R^{c}_{j,k} = Np1,p2 \,.\, Np2,k \;/\; Np1,j \tag{16}$$

then the two velocity ratios are related by

$$R^{c}_{j,k} = 1 \,/\, R^{c}_{j,k} \tag{17}$$

also

$$R^{c}_{j,k} = (Np1,j\text{-}Np1,k) \,/\, (Np1,i\text{-}Np1,k) \tag{18}$$

$$R^{c}_{j,k} = (Np1,j\text{-}Np1,p2 \,.Np2,k)/(Np1,i\text{-}Np1,p2 \,.\, Np2,k) \tag{19}$$

From a single nomograph such as the one shown in Figure 8, all of the thirty six velocity ratios of a five coaxial links FGE or EGM can be easily found.

4.4 Kinematic of a triple-planet FGE

Figure 10 shows an FGE having three planets in a chain. Note that the FGEs shown in Figures 5, 7 and 10 can themselves serve as EGMs since they have more than five coaxial links at the first level.

A nomograph can be drawn to this FGE in the same manner as before. The gear ratios for the coaxial gears that are not meshing directly with the first planet gear on which the nomograph will be drawn and are meshing with the second planet gear can be found from equation (10) in terms of the gear ratios of the first and second planets ($N_{p1,p2}$), while the gear ratios for the coaxial gears that are meshing with the third planet gear can be found in terms of the gear ratios of the first and second planets ($N_{p1,p2}$) and the second and third planets ($N_{p2,p3}$) from

$$N_{p1,x} = N_{p1,p2} \,.\, N_{p2,p3} \,.\, N_{p3,x} \tag{20}$$

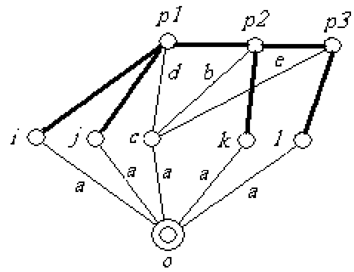


Figure 10. Graph of an FGE having three planets in a chain.

FGEs and EGMs having more than two planets in a chain are not practical and will be excluded from further consideration.

4.5 Overall Velocity Ratio Analysis or Unification of FGEs

To find the overall velocity ratios $R^{z}_{y,x}$ for an EGM, we unify the nomographs of the FGEs in one nomograph called the system nomograph, Figure 11. We first unify FGEs that are connected to each other by two common links i.e. they have two links with the same labeling. Whenever it is possible, unify one of the resulting subsystems with one of the remaining FGEs or with one of the other resulting subsystems until all the FGEs and subsystems are unified in one system.

Since the two FGEs share two common links, therefore these links must have the same angular velocities. Let b1 and b2 be the common links, p1 be the first planet of the first FGE to be unified, y be any link from the second FGE other than b1 and b2, then by using link b1 as a bridge, we express the overall velocity ratio of the unified subsystem in terms of two of the velocity ratios of the FGEs as

$$R^{b2}_{y,p1} = R^{b2}_{y,b1} \,.\, R^{b2}_{b1,p1} \tag{21}$$

where $R^{b2}_{b1,p1}$ is associated with the first FGE and $R^{b2}_{y,b1}$ is associated with the second FGE. From the unified nomograph, we can write

$$R^{b2}_{y,p1} = (Np1,y\text{-}Np1,b2) \,/\, (1,i\text{-}Np1,b2) \tag{22}$$

where **N***p1,y* is the distance between the first FGE carrier and link y axes. For a particular EGT, two nomographs are drawn in Figure 12. Since N*p1,b2* is equal to zero for this train, applying Eq. (22) yields

$$R^{b2}_{y,p1} = Np1,y/1 \tag{23}$$

Combining equations (21) and (23) yields

$$Np1,y = R^{b2}_{y,b1} \,.\, R^{b2}_{b1,p1}$$

For a system having more than two FGEs, the unification process continues between FGEs or / and unified subsystems until the highest-level system becomes the required mechanism. This way, a system nomograph can be obtained in terms of the gear ratios of its gear pairs.

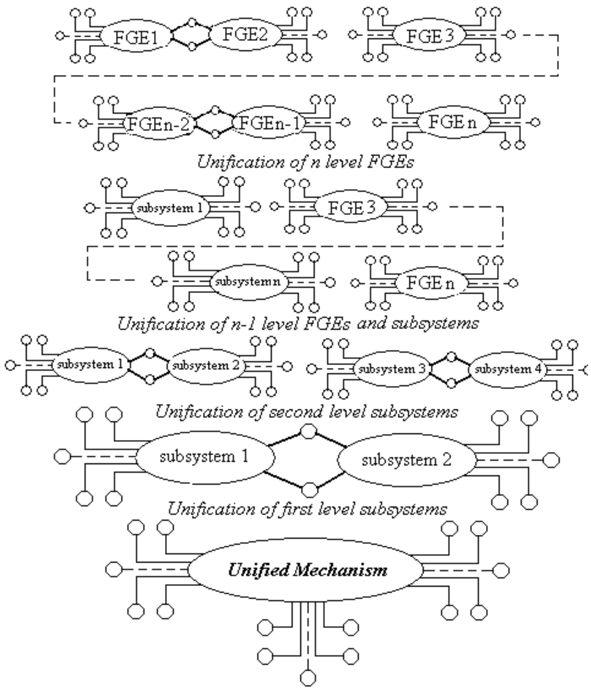
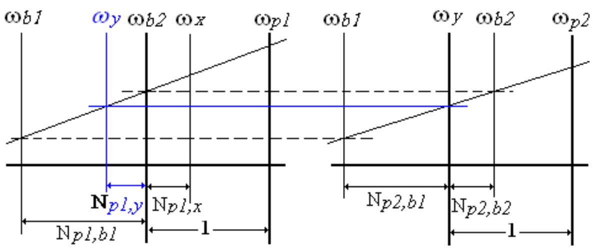


Figure 11. Unification of the FGEs of a system



(a) First FGE (b) Second FGE

Figure 12. Unification of two FGE nomographs into one nomograph. (a) Nomograph for the first FGE (b) Nomograph for the second FGE or/and unified EGT nomograph.

The reliability of the method will be established by applying it to four transmission gear trains for which solutions are fully available in the literature.

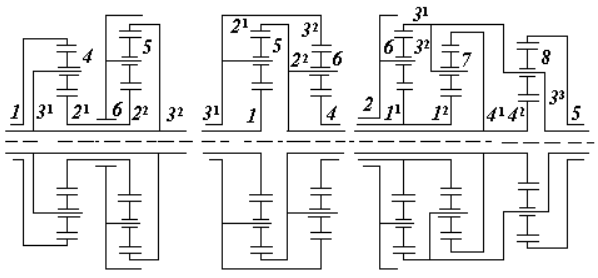
4.6 Nomographs of Some Commonly Used Compound Epicyclic Gear Trains

Figure 13 shows three commonly used Epicyclic Gear Trains which will be used to demonstrate this methodology.

Simpson gear train

Figure 13(a) shows the Simpson gear train which is composed of two single planet FGEs. Figure 12 shows the corresponding nomographs for these FGEs and their unification process. Figure 14 shows the unified system nomograph.

$$N5,3^2 = Z5 / Z3^2 \quad (25)$$
$$N5,2^2 = - Z5 / Z2^2 \quad (26)$$
$$N4,1 = Z4 / Z1 \quad (27)$$
$$N4,2^1 = - Z4 / Z2^1 \quad (28)$$



(a) (b) (c)

Figure 13. Some commonly used compound epicyclic gear trains. (a)Simpson gear train (b)Type-6206 gear train (c) Type-8001gear train

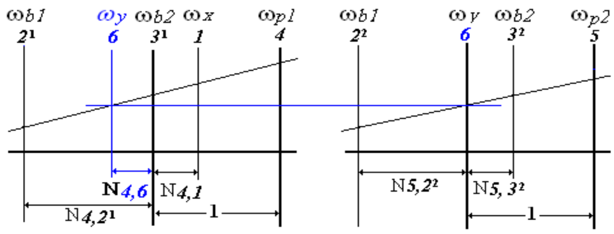


Figure 14. System nomograph for the Simpson gear train shown in Figure 13(a)

From this nomograph twenty-four overall velocity ratios can be obtained by observation as follows. From eq. (21)

$$R^{3}_{6-4} = R^{3}_{6-2} . R^{3}_{2-4} \quad (29)$$

From the first FGE nomograph

$$R^{3}_{2-4} = N4,2^1 \quad (30)$$

And from the second FGE nomograph

$$R^{3}_{6-2} = - N5,3^2 / (N5,2^2 - N5,3^2) \quad (31)$$

Substituting Eq. (30) and Eq. (31) into Eq. (29), we get

$$R^{3}_{6-4} = - N5,3^2 . N4,2^1 / (N5,2^2 - N5,3^2) \quad (32)$$

But from the unified nomograph

$$R^{3}_{6-4} = \mathbf{N}4,6 \quad (33)$$

Therefore,

$$\mathbf{N}4,6 = - N5,3^2 . N4,2^1 / (N5,2^2 - N5,3^2) \quad (34)$$

also $R^{3}_{1-2} = N4,1 / N4,2^1$ (35)

$$R^{3}_{1-6} = N4,1 / \mathbf{N}4,6 \quad (36)$$

and $R^{6}_{1-2} = (N4,1 - \mathbf{N}4,6) / (N4,2^1 - \mathbf{N}4,6)$ (37)

This completes the overall velocity ratio analysis for the Simpson gear train.

Type 6206 Gear Train

Figure 13(b) shows type 6206 gear train which is composed of two single planet FGEs. Figure 15 shows the corresponding nomographs for these FGEs and their unified system nomograph.

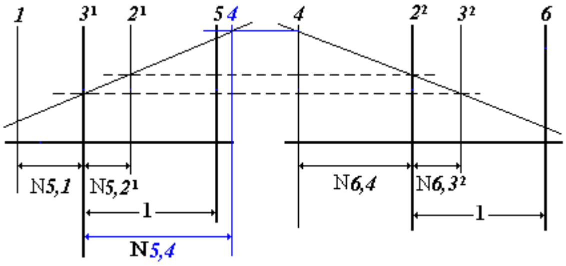


Figure 15. Nomographs for the FGEs of type 6206 gear train and their unified system nomograph.

$$N6,3^2 = Z6/Z3^2 \tag{38}$$

$$N6,4 = -Z6/Z4 \tag{39}$$

$$N5,2^1 = Z5/Z2^1 \tag{40}$$

$$N5,1 = -Z5/Z1 \tag{41}$$

From eq. (21)

$$R^{2}_{4-5} = R^{2}_{4-3} \cdot R^{2}_{3-5} \tag{42}$$

From the first FGE nomograph

$$R^{2}_{3-5} = -N5,2^1 / 1 - N5,2^1 \tag{43}$$

And from the second FGE nomograph

$$R^{2}_{4-3} = N6,4 / N6,3^2 \tag{44}$$

Substituting Eq. (43) and Eq. (44) into Eq. (42), we get

$$R^{2}_{4-5} = -N5,2^1 \,.\, N6,4 / N6,3^2 \,(1 - N5,2^1) \tag{45}$$

But from the unified nomograph

$$R^{2}_{4-5} = (\mathbf{N}5,4 - N5,2^1)/(1 - N5,2^1) \tag{46}$$

Therefore, from Eq. (45) and Eq. (46)

$$\mathbf{N}5,4 = N5,2^1\,(1 - N6,4 / N6,3^2) \tag{47}$$

$$\text{also } R^{4}_{1-2} = (N5,1 - \mathbf{N}5,4) / (N5,2^1 - \mathbf{N}5,4) \tag{48}$$

And so on for the other overall velocity ratios. This completes the overall velocity ratio analysis for type 6206 gear train.

Type-8001 Gear Train

Figure 13(c) shows type-8001 gear train which is composed of three single planet FGEs. Figure 16 shows the unification process for this train. Figure 17 shows the corresponding nomographs for the first and second FGEs of this train and their unified subsystem nomograph.

$$R^{3}_{4-6} = R^{3}_{4-1} \,.\, R^{3}_{1-6} \tag{49}$$

$$R^{3}_{4-6} = N7,4^1 \,.(N6,1^1 - N6,3^1) / N7,1^2\,(1 - N6,3^1) \tag{50}$$

$$R^{3}_{4-6} = (\mathbf{N}6,4^1 - N6,3^1) / (1 - N6,3^1) \tag{51}$$

$$\mathbf{N}6,4^1 = N6,3^1 + N7,4^1 \,\mathbf{.}\, (N6,1^1 - N6,3^1) / N7,1^2 \tag{52}$$

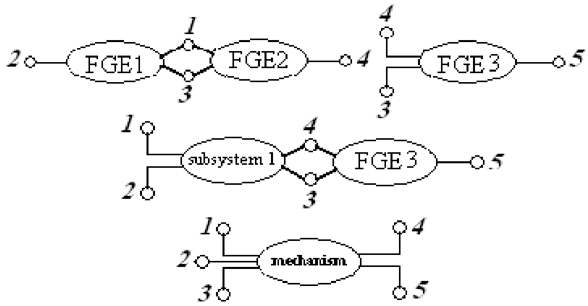


Figure 16. The unification process for type-8001 gear train

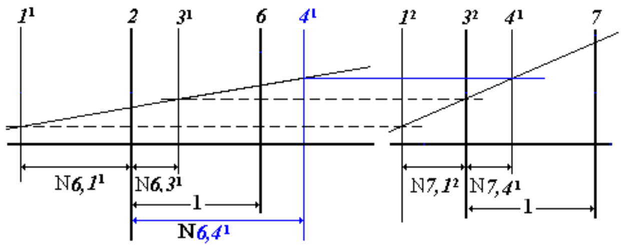


Figure 17. Nomographs for the first and second FGEs of type-8001 gear train and their unified subsystem.

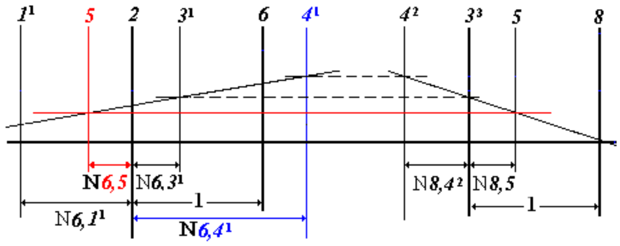


Figure 18. Nomographs for subsystem 1and FGE 3 of type-8001 gear train and their unified system nomograph.

$$R^{3}_{5-6} = R^{3}_{5-4} \,.\, R^{3}_{4-6} \tag{53}$$

$$R^{3}_{5-6} = N8,5 \,.\, (\mathbf{N}6,4^1 - N6,3^1) / N8,4^2\,(1 - N6,3^1) \tag{54}$$

$$R^{3}_{5-6} = (\mathbf{N}6,5 - N6,3^1) / (1 - N6,3^1) \tag{55}$$

$$\mathbf{N}6,5 = N6,3^1 + N8,5 \,\mathbf{.}\, (\mathbf{N}6,4^1 - N6,3^1) / N8,4^2 \tag{56}$$

The unified system nomograph for subsystem 1 and FGE3 of Type-8001 is given in Figure 18. This completes the overall velocity ratio analysis for type-8001 gear train.

Example

Figure 19 shows the schematic diagram, canonical graph representation, and the FGEs of a compound EGM. The speed ratio analysis starts by unifying FGEs one and two through links one and five, as shown in Figure 20. From Eq. (10), we can write

$$N7,5^1 = N7,8 \,.\, N8,5^1 \tag{57}$$

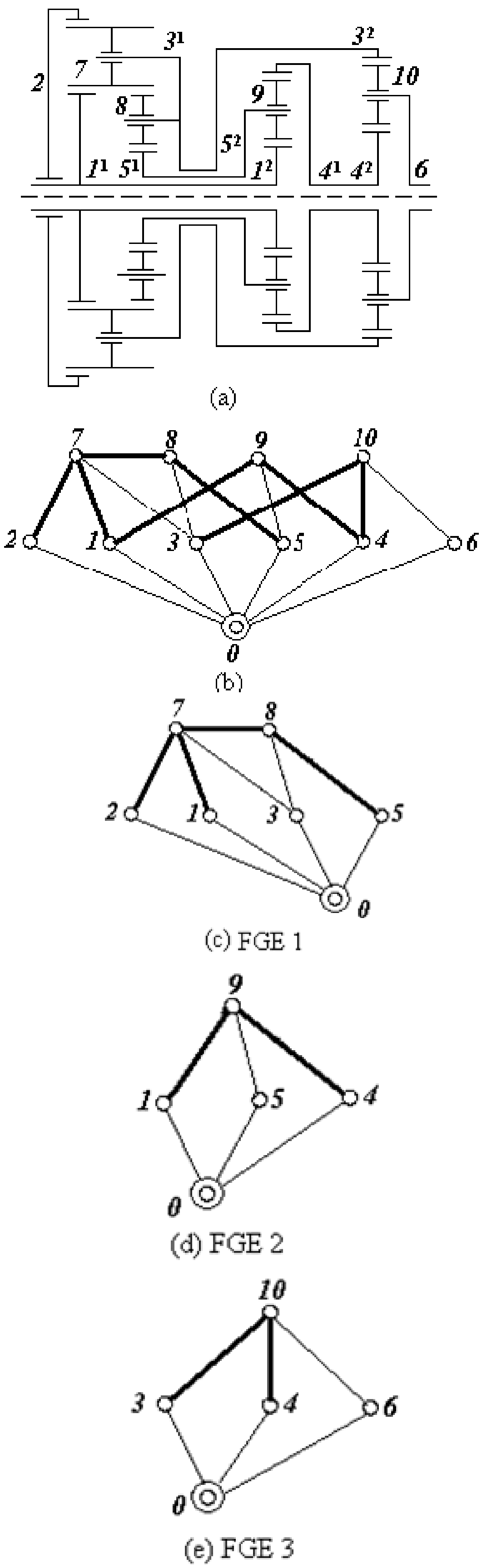


Figure 19. (a) schematic diagram, (b)canonical graph, (c) through (e) FGEs of a compound EGM.

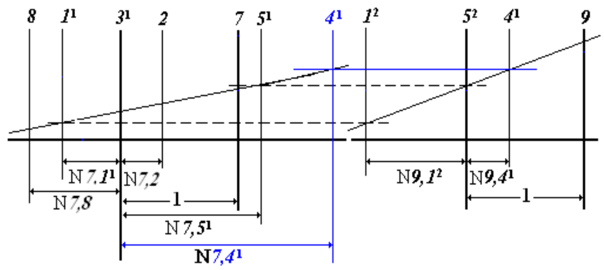


Figure 20 Unification of FGE one and two.

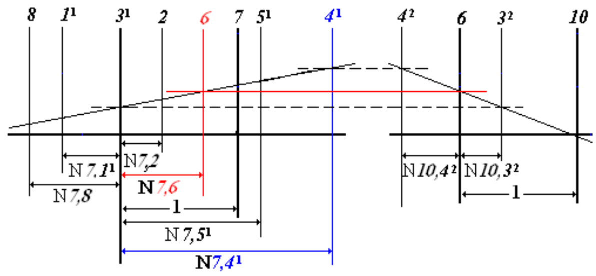


Figure 21. Unification of subsystem one and FGE three.

Subsystem one contains FGEs one and two. Then, unifying subsystem one with FGE three through links three and four, as shown in Figure 21, one can obtain the final system kinematic nomograph. From this kinematic nomograph all of the velocity ratios can be obtained by observation as before.

5. CONCLUSIONS

This paper contributes to the development of a new method for the derivation of the velocity ratio of an EGM. The well established graph theory is used to represent the system and then to detect the FGEs. The identification of FGEs leads to automated construction of the kinematic nomographs in a systematic manner.

The main advantage of a nomograph is its simplicity. Also, algebraic expressions for all of the velocity ratios of an FGE or/and an EGM can be easily obtained by observation without mathematically manipulating the fundamental circuit equations and without specifying the exact size of each gear. In addition, it provides a better insight of the effects of the FGEs and their gear sizes on the overall velocity ratio of an EGM. This is very helpful in the identification of a clutching sequence during the design phase of an epicyclic-type transmission mechanism. The reliability of the method has been established by applying it to four transmission gear trains for which solutions are fully available in the literature. The method is also applicable to the kinematic analysis of bevel-gear trains. Future work will apply this method to the development of an automated methodology for the enumeration of various clutching sequences associated with an epicyclic gear mechanism.

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