VALIDATION OF THE STRENGTH PROPERTIES OF AVOCADO PEAR USING DISCRETE ELEMENT MODELING

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(Received December 2008 and accepted June 2009)

The use of discrete element modeling for the study of prediction of bruises and damages of soft agricultural particulates has been very limited. This is because the discrete element computations become unstable for very soft object and may require extreme computation time as a result of short time step required. This study investigates the use of a new non-linear damping relationship combined with the non-linear Hertz contact model for soft viscoelastic materials. It is part of further studies on the validation of Discrete Element Model on some practical handling operations of agricultural materials, specifically fruit handling and transportation. The model was validated using experimental reports on the strength properties of avocado pear after 0 to 20 days of harvest dropped from heights ranging from 100 to 500mm. The trials show prediction of stresses at all drop heights for all days of harvest close to the experimental yield stress which indicates failure (cracking and crushing). However, the predictions fitted better for very soft products i.e. those over 10 days of harvest. This shows that the new non-linear damping relationship is suitable and promising for modeling the behaviour of soft agricultural particulates.

Keywords: Discrete Element Method, Validation, Handling, Strength Properties, Fruit.

1. INTRODUCTION

Avocado pear (Persea americana) is a fruit obtained from an evergreen tree which is native to tropical America but now formally produced for local consumption in most African countries. For the past few years, however, they have become increasingly important in many tropical and sub-tropical regions in Africa due to increasing numbers of hotels and restaurants and export⁴ while Kenya and Dominican Republic are the two leading exporters of avocado pear to the EU with 7.6 and 0.8% of their total import⁵. The effect of bio-chemical and physical agents on deterioration before and after harvesting which eventually affects the strength properties of the fruit have been discussed³, ⁴. The deterioration of fruits generally after harvesting are dependent on many factors, some of which include the impact damage from method and equipment used in harvesting. The bruises or damages in form of compressed portions are potential points for deterioration as they become breeding places for microorganism or suitable points for bio-chemical reactions.

The use of Discrete Element Modeling (DEM) for soft agricultural particulates has not been fully explored. Most of the reported works have been limited to flow and movement of hard particles. However the DEM has been used for the study of prediction of bruises and damages as a result of impact on apple⁴, bruise prediction and vibration damage of apple during transport⁵ and deformation in viscoelastic material⁶. Reports of the impact force, velocity at impact and other information at the contact point were predicted using a model developed based on the DEM⁶, ⁸, ¹² with a view to obtaining the depth of damage and provide insight into prediction of safe height of drop and padding surfaces for containers, belts and other surfaces where necessary. These results were purely theoretical with no practical validation.
DE, the computation for very soft object requires very low time step under high compressive stress. Each calculation step applies the Newton’s laws of motion to all particles and walls and uses appropriate non-linear viscoelastic force-displacement law at each contact. The contact law uses a combination of elastic and viscous (damping) contact relationship based on the Hertz-Mindlin equations for interactions at the contact point to compute the force (normal and tangential) for the elastic and damping component, displacement as well as the mass/gravitational pull components.

Contact Theory

The interaction at the contact between two particles is commonly envisaged or modelled using the simplest rheological damping model, the Kelvin-Voigt model. The equations that govern the linear system for spring and dashpot assume a linear spring and dashpot in parallel with

$$F_s = -C_n x \quad \text{and} \quad F_d = -D \frac{dx}{dt} = -D \dot{x}$$

(2.1)

where $F_s$ is the spring (elastic) force, $F_d$ is the viscous damping force, $C_n$ is the spring stiffness, $D$ the damping coefficient and $x$ is the displacement of the particle from the initial position (the approach of the two centres for two spherical bodies in contact) while $\dot{x}$ is the first derivative of $x$ with respect to time (the oscillation or relative velocity). In most DE simulations the $C_n$ and $D$ are selected and supplied. However the Hertz theory which predicts a non-linear relationship has been found to be more realistic for the elastic component with

$$F_n = K \frac{R_1 R_2}{R} \alpha^{\frac{1}{2}}$$

(2.2)

where $F_n$ is the normal force, $K$ is material stiffness, $R$ is the geometric parameter (equivalent radius) and $\alpha$ is the deformation taken as the approach of the two objects under load. Further details about the calculation of the equivalent radius from the radii and types of the contacting bodies (flat, convex and concave surfaces) as well as the material stiffness properties have been discussed in other reports.$^{[12, 15, 21]}$

The stiffness is obtainable dynamical

$$C_n = \frac{dF_n}{d\alpha} = \frac{3}{2} K \frac{R_1 R_2}{R} \alpha^{\frac{1}{2}}$$

(2.3)

Similarly, the linear damping relation has been found$^{[22]}$ to be unrealistic and authors of reference$^{[22]}$ obtained a non-linear relation in their investigation. The relation is a function of both displacement and velocity as opposed to the commonly used linear equation which is only a function of velocity. This type of damping gives a more realistic relation compared to the linear model$^{[22, 23]}$. The general relationship gives

$$F_d = -\dot{x} \sqrt{\frac{5}{6} \nu \sqrt{C_n m \alpha}}$$

(2.4)
where the damping ratio \( \psi \) is dependent on the coefficient of restitution \( e \), with
\[
\psi = -\frac{\ln e}{\sqrt{\ln e^2 + \pi^2}}
\]
(2.5)
and \( m^* \) is a function of the mass of the bodies. For two objects 1 and 2 in contact \( m^* \) is obtained as
\[
m^* = \frac{m_1m_2}{m_1 + m_2}
\]
(2.6)
This also implies a dynamic calculation of damping constant which is dependent on the material properties.

This produces a damping force for non-linear hysteretic system where residual deformation is predicted after unloading. In DEM the commonly used damper is the global damping which is the damping applied equally to all particles for energy dissipation to bring the system to equilibrium during consolidation or deposition process because the normal force is taken to be elastic. It is often a dashpot connecting each particle to the ground\(^{[24]} \) and this method is retained in most subsequent DE models. The non linear relationships are suitable and appropriate for the soft (low modulus) agricultural and biomaterials which are viscoelastic and have residual deformation even under self weight.

In the hertz theory the contact area is a circular area and the pressure distribution over the contact area is found to be represented by a hemisphere with a maximum at the centre. The general equation for normal stresses and also for the radial and tangential stresses (not calculated by Hertz) on a circular region of radius “a” have been derived\(^{[23]} \), hence
\[
\sigma_n = -P_{\text{max}} \left(1 - \frac{r^2}{a^2}\right)^{\frac{1}{2}}
\]
(2.7)
where \( r = 0 \) at the centre and \( r = a \) at the edge of the contact circle. The tangential and radial stresses are minimum (negligible fraction of \( P_{\text{max}} \)) at the centre while their maximum values occur at the boundary surface of the circle with
\[
\sigma_t = \sigma_r = \pm P_{\text{max}} \left(1 - \frac{2v}{3}\right)
\]
(2.8)
where \( v \) is the Poisson ratio. Using these relationships and computing the contact area using the Hertz theory\(^{[12]} \) the stresses at contact were obtained during the cycles.

The model has the capability to handle vertical, horizontal and inclined as well as stationary and moving walls. This implies that containers of various shapes can be modeled and with particle interaction within the containers. Modeling them as hard and rigid particles can only produce the displacement profile but the appropriate deformation at contact points is always limited to ensure free movement. For this study both the motion and deformation are very important. There is no limit to the overlap at contact in this model (the deformation). The model is capable of redistributing the deformed portion resulting in increase in size of the deformed particle hence the porosity of the medium as the materials are loaded can be obtained\(^{[16]} \).

### Droplet Simulation

The model was used to simulate the drop impact of avocado pear for five heights of fall as performed experimentally\(^{[11]} \) at 100, 200, 300, 400 and 500 mm. A spherical shape was assumed, as the experimental work used in the comparison with the predicted data dropped falling pears with wide and spherically-shaped lower part. The properties and other parameters for avocado pear used in the experiment\(^{[1]} \) and in the simulation are presented in Table 1. Preliminary trial runs using the method of Raji and Favier\(^{[26]} \) were performed to determine the appropriate coefficient of restitution and resulting damping ratio for avocado pear. An elastic modulus of 200MNm\(^{-2}\) which is the average from the list presented in Table 1 was used. The damping ratio used in this code is dependent on the coefficient of restitution and is not based on an arbitrary selection, as used by other researchers\(^{[10, 26, 27]} \). In most research works, damping ratio is just mentioned but the actual value and how it was obtained for the material is often unreported. Also, in this work, the stiffness constants were calculated dynamically but not chosen or selected as practiced in the simulation for hard particles.

The rebound profile, forces, and stresses at impact(s) as predicted by the model were recorded and reported. These were compared with the experimental results\(^{[1]} \).

<table>
<thead>
<tr>
<th>Time of Harvest (Days)</th>
<th>Elastic Modulus (kNm(^{-2}))</th>
<th>Rigidity Modulus (Calculated) (kNm(^{-2}))</th>
<th>Yield Stress (kNm(^{-2}))</th>
<th>Rupture Stress (kNm(^{-2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>480024</td>
<td>184625</td>
<td>1020</td>
<td>858</td>
</tr>
<tr>
<td>5</td>
<td>325110</td>
<td>125042</td>
<td>752</td>
<td>592</td>
</tr>
<tr>
<td>10</td>
<td>115007</td>
<td>44234</td>
<td>244</td>
<td>184</td>
</tr>
<tr>
<td>15</td>
<td>73550</td>
<td>28288</td>
<td>165</td>
<td>105</td>
</tr>
<tr>
<td>20</td>
<td>48115</td>
<td>18505</td>
<td>103</td>
<td>76</td>
</tr>
</tbody>
</table>

### Validation of Damping Theory

The results of preliminary simulations performed to determine the appropriate damping ratio for avocado pear, since it was not part of the parameters available from the experimental report, are as presented in Figure 1 (a – c). These also serve as the validation process for the model damping component.

From Figure 1(a – c) showing the Force-Time (F-T) curves of impact of a pear with elastic modulus of 215kNm\(^{-2}\) dropped from a height of 100mm onto a hard surface, a peak force of 270N was predicted as shown in Figure 1. The pattern of variation for the resultant, normal and damping forces are as shown in

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**Table 1: Properties of Avocado pear with diameter 80 ±5mm and mass of 75 ±10g\(^{[1]} \)**

**3. RESULTS AND DISCUSSION**

**Validation of Damping Theory**

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Figures (a – c) during the first impact. The pear was predicted to have bounced back and return to the surface for the second impact as shown in Figure 1b with a peak force of 15N on impact. Because of the low coefficient of restitution, the pear did not disengage from the surface again but undergoes what can be termed as vibrations on the surface and came to rest as shown in Figure 1c with the force of contact being 0.748N (i.e. 74.8g). It should be noted that Figure 1 (a through c) is taken from the same graph for the drop impact, but separated and enlarged here to show different stages during the processes of rebound and coming to rest.

The damping component, as can be seen from the figures, ease out the process of upward movement with an elongated portion to the right. Although the peak force was 270N but the real peak force which produces the eventual escape velocity is the mini peak force of approximately 20N on the curve (second curve) for the resultant force in Figure 1a. This slows down the object hence escaping with a lower speed[12] (the next impact comes with a lower impact force). On the second impact, the impact force of 15N was damped also to a lower mini peak force of approximately 1N (Figure 1b) and the object could not escape from the surface but rather made to ‘stick’ to the surface and undergo a short term vibration before coming to rest with a constant contact force equal to the weight of the object. This behaviour reflects better the behaviour of the viscoelastic agricultural materials as opposed to the normal linear damping often used for other engineering materials in DE simulations[5, 15, 16].

Drop Impact
The results for the drop impact with coefficient of restitutions (e) of 0.1 to 0.6 are as presented in Figure 2. The coefficient of restitution for pear was not determined experimentally and not reported in the literature. Therefore preliminary investigation was done to find out which coefficient of restitution produced the best behavior closest to what is obtained for fruits. This led to the selection of the appropriate values to be used for further simulation. The predicted rebounding and impacts with the peak forces and the values of the ‘mini’ peak are as presented. It was observed that, in all, the last two impacts for each are same, as the object sticks to the surface and vibrates to rest with a second peak force. It then settles to a contact force equal to the weight of the object.

The number of rebounds recorded for e higher than 0.2 are not realistic for fruits. A fruit dropped, impacting and rising to impact several times, will create a number of bruised surfaces or points which becomes potential points for deterioration. In reality the nature of the products does not allow for continuous vibration (rebounding) as they are not purely elastic. Also the longer time taken to come to rest for e greater than 0.2, as shown in Table 2, are not realistic for most soft agricultural products.

With an e of 0.1 (ψ = 0.59), the pear impacted and disengaged faster than others. Therefore an e of 0.1 which gives a single rebound before coming to rest was chosen for further studies on this simulation.
Although it gave a relatively higher impact force but the ability to closely represent the behaviour of the fruit is of more importance. In addition the result above shows that the contact laws and the new damping relationship are more reliable than the linear relationship often used.

The results of the model prediction of the drop impact for the pears at 0, 5, 10, 15 and 20 days after harvest, dropped from a height of 100, 200, 300, 400 and 500mm, are as presented in Figures 3 and 4. The force and stress predicted by the model are presented and compared with yield and rupture stress (Table 1) reported experimentally\cite{1} for the corresponding days. This is with a view to obtaining the critical drop height for the fruit on the days after harvest.

Table 2: Impact parameters with different damping ratios

<table>
<thead>
<tr>
<th>Damping Ratio</th>
<th>Coefficient of Restitution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>Time of first impact, 10^{-3}s</td>
<td>110.77</td>
</tr>
<tr>
<td>Time taken to come to rest 10^{-3}s</td>
<td>135.63</td>
</tr>
<tr>
<td>Damped peak force during the first contact, N</td>
<td>25.7</td>
</tr>
<tr>
<td>Number of Bouncing</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3 predicted closely the findings of the laboratory investigations. The model predictions show that failure occurred at heights which are higher than the heights obtained for bruising during experiments for all the days after harvest. All the heights predicted are close to or same as the height obtained for cracking in the experimental report except for day 0. This may be as a result of the fact that the fruit was still very hard and the model was developed to handle soft products. This also shows that the viscous effects need to be modified for some ranges of hardness.

Figure 2. Peak forces and number of impact at different damping ratios

The trend of the variation of the forces of impact at each height of drop is presented in Figure 3. The force of impact increases with the height of drop for all the days. This is in agreement with the predictions reported and validated for apples modeled as spherical objects\cite{5,6}. However experimental validation for avocado pear is not done for this study since the model used has been validated for another fruit modeled as spherical object in a similar way done in this study.

The results (Figure 4) when compared with the outcome of the experimental findings\cite{3} presented in
Table 3: Critical heights for damage

<table>
<thead>
<tr>
<th>Days after harvest, day</th>
<th>Critical height for damage, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Scuffing $^{[1]}$</td>
</tr>
<tr>
<td>0</td>
<td>98</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
</tr>
<tr>
<td>10</td>
<td>27</td>
</tr>
<tr>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
</tr>
</tbody>
</table>

The heights of drop were also confirmed with the stresses at impact. These are presented in Figure 4. For days 0 and 5 after harvest (Figure 4), the model predictions show that dropping from heights of 100 and 200mm resulted in stresses below the experimental yield stresses (1020 and 752 kNm$^{-2}$)$^{[1]}$ while only drop height 100 resulted in a stress lower than rupture stresses (858 and 592 kNm$^{-2}$) for the two days. The critical heights predicted are 250 and 198 respectively for the days (0 and 5). The model predicted a stress (259.53 kNm$^{-2}$) which is higher than the yield stress (244 kNm$^{-2}$) for day 10 at all the drop heights but a very close stress to the yield stress was predicted at drop height of 100mm. This indicates that a height slightly below 100mm was predicted for 10 days after harvest. This was found to be 98mm. The predictions for days 15 and 20 showed that the fruit will be damaged at all the heights studied (predicted stresses are higher than the yield stresses), the critical heights are thus far below 100mm.

4. CONCLUSIONS

The damping relationship used and tried in this study is more realistic for soft deformable agricultural materials. The relationship predicted the failure criteria at lower yield stresses better than for the hard products. This shows that it is suitable for predictions of soft deformable materials. Further studies on the loading, compression and conveying damages will be required for industrial applications of this model.

Acknowledgement

The authors acknowledge the support from the John D. and Catherine T. MacArthur Foundation/University of Ibadan Staff Development Grant and the Department of Agricultural and Bioresource Engineering, University of Saskatchewan, Saskatoon, Canada.

Nomenclature

- $a$: Radius of contact area, m
- $C_n$: Spring stiffness
- $D$: Damping coefficient
- $F_n$: Normal force, N
- $F_s$: Spring (elastic) force, N
- $F_d$: Viscous damping force, N
- $K$: Material stiffness
- $m_*$: A function of the mass of the bodies
- $m_1$ and $m_2$: mass of two objects 1 and 2 in contact, g
- $P$: Pressure, Nm$^{-2}$
- $r$: Radius, m
- $R$: Geometric parameter (equivalent radius)
- $x_i$: Displacement of the particle from the initial position (the approach of the two centers for two spherical bodies in contact)
\( \dot{x} \), The first derivative of \( x \) with respect to time (the oscillation or relative velocity).
\( \alpha \) Deformation taken as the approach of the two objects under load.
\( e \) Coefficient of restitution
\( \psi \) Damping ratio
\( \sigma \) Stress, N/m²
\( v \) Poisson ratio.

REFERENCES