THREE DIMENSIONAL ROTATING HYDROMAGNETIC TRANSIENT CONVECTION FLOW OF LIQUID METAL IN A POROUS MEDIUM WITH HALL AND IONSLIP CURRENT EFFECTS

S.K. Ghosh1, O.A. Beg2, S. Rawat3, T.A. Beg4

1Applied Mathematics Program, Department of Mathematics, Narajole Raj College, West Bengal, Indi, E-mail: g_swapan2002@yahoo.com
2Mechanical Engineering Department, Sheffield Hallam University, England, UK. Email:O.Beg@shu.ac.uk and docoanwarbeg@hotmail.co.uk
3Centre de Mise en Forme des Materiaux(CEMEF), 1 Rue claude Daunesse, Franc, E-mail: sam.rawat@gmail.com
4Engineering Mechanics and Renewable Energy Systems Research, Manchester, England, UK., Email: tsab001@googlemail.com

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A mathematical model to study the transient rotating magnetohydrodynamic (MHD) free convection from a vertical surface embedded in a liquid-metal saturated Darcian porous medium, is presented. The governing nonlinear coupled partial differential equations of transport are transformed to a system of dimensionless coupled differential equations in terms of time (t) and z-coordinate. The dimensionless momentum and energy equations are solved analytically under physically realistic boundary conditions. The variations of the three dependent flow variables (primary velocity u, secondary velocity v, and dimensionless temperature $\theta$) with Grashof number i.e. free convection parameter, Er (rotational parameter), Da (Darcy number), Nm (hydromagnetic number), $\beta_i$ (ionslip parameter) and Prandtl number are presented graphically and discussed in detail. Very low Prandtl numbers are considered since these are representative of liquid metals which possess very high thermal conductivities. Solutions are also obtained for the purely fluid case for which Da $\rightarrow \infty$. Both cases of heating of the plate by free convection currents (Gr < 0) and cooling of the plate by free convection currents (Gr > 0) are considered. The model finds applications in geophysical regimes and also magneto-energy systems and industrial materials processing exploiting magnetic fields.

Keywords: Mathematical model, hydromagnetics, non-Darcy, rotating fluid, Hall currents, Ionslip currents, Porous Medium, MHD energy systems.

1. INTRODUCTION

Magneto-fluid dynamic flows with heat transfer have attracted widespread interest from astrophysical, renewable energy systems and also hypersonic aerodynamics researchers for a number of decades. Many studies have been presented to analyze the effects of magnetic fields on flow fields and convective flow regimes. An early study was presented by Rossow[1]. Researchers later considered the more complex effects of Hall currents on MHD (magnetohydrodynamic) convection flows. In partially-ionized fluids (e.g. water solution seeded with Potassium) which occur in MHD energy systems and accelerators[2], Hall and ionslip currents become significant. The presence of longitudinal Hall currents in a flow creates a transverse body force which can lead to transverse pressure gradients, velocity gradients etc[3]. Much attention has been directed at elucidating a range of MHD flows with Hall current and ionslip effects, to provide a more realistic model of energy systems as well as geophysical flow phenomena. Pop[4] presented an analysis of the magnetohydrodynamic convection in the vicinity of an
accelerated plate incorporating the effects of Hall currents. Cramer and Paj[5] have indicated the importance of Hall current effects in Hartmann channel flows. Masapati et al[6] studied the influence of Hall and ionslip currents on hydromagnetic entry channel flow. Significant studies of heat transfer in MHD flows with Hall currents have also been reported since the 1970s. Bhat and Mittal[3] later investigated the heat transfer regime with uniform wall heat flux in the developing flow region in the presence of Hall and ionslip currents. Bhat and Mittal[8] further studied the laminar forced convective MHD channel heat transfer problem with Hall and ionslip currents. Soundalgekar et al.[9] computationally studied the hydromagnetic Couette flow and convection with Hall and Ionslip currents. The case of thermal convective MHD channel flow with Hall currents was examined by Bhat and Mittal[7] who modelled the constant wall flux scenario. Singh[10] presented one of the first papers on transient MHD flow with Hall currents for the Stokes problem past an infinite porous vertical surface. The free convection MHD flow with Hall currents were studied by Niranjan et al.[11] for a horizontal channel and by Takhar and Ram[12] for a viscous heat- generating fluid with oscillating wall temperature. A full numerical solution to the MHD convective Stokes problem with Hall currents was presented by Takhar et al.[14]. Dash and Das[15] considered the coupled effects of lattice mass flux [suction/blowing] and internal heat generation with Hall currents on hydromagnetic convection along an accelerated vertical surface. The case of hydromagnetic heat transfer past a stretching surface with Hall currents was studied numerically by Gupta et al.[16]. Bhargava and Takhar[17] analysed the more complex case of second order fluid MHD convection in a parallel porous plate configuration with Hall currents. The transient MHD convection of a second order viscoelastic fluid with Hall effects was investigated more recently by Hayat et al.[18]. Elshehawey et al.[19] presented a numerical analysis of the combined effects of thermal conductivity variation and Hall and ionslip currents on MHD convection flow. Macheret al.[20] investigated the importance of Hall and ionslip current dynamics on the control of high Mach number aerodynamic flows.

In all of the above studies, neither rotational nor porous media hydrodynamic effects were studied. Rotating MHD flows are of considerable importance, owing to applications in geophysics and also materials processing systems. For example, Ghosh et al.[21] studied thermal radiation effects on rotating hydromagnetic gas convection. Several authors have considered the influence of Hall currents in rotating MHD flows. Ram and Takhar[22] used a finite difference scheme to study the two-dimensional hydromagnetic natural convection heat transfer from an infinite vertical surface in a rotating channel regime with Hall current effects. Takhar et al.[23] considered the Hall current effects on transient dusty magnetofluid flow in a revolving channel configuration. The combined effects of oscillating wall temperature, Hall and ionslip currents on rotating convection were analysed by Ram et al.[24]. Takhar and Jhu[25] examined the hydromagnetic flow past an impulsively started surface in a rotating system with Hall and ionslip currents. Takhar et al.[26] extended this analysis to resolve numerically the effects of free stream velocity and Hall and ionslip currents on MHD flow past a translating plate. Naroua et al.[27] studied combined heat source and Hall/ionslip current effects on rotating unsteady magnetohydrodynamic convection flow.

As indicated earlier, porous media effects have not been incorporated in any of these studies. In many industrial applications, porosity of materials is an intrinsic aspect of the engineering process. Ceramics, batch reactors, metallic foams and filtration systems all utilize porosity. In geophysical systems, the porosity of soils can exert a considerable influence on flow and temperature profiles. An excellent survey of general applications of porous media fluid dynamics and transport processes with heat transfer is available in Kaviany[28]. In addition, the Coriolis force due to the earth’s rotation involves rotational body forces, and the earth’s magnetic field induces hydromagnetism. It is therefore of great interest to study hydromagnetic rotating flows in porous media, particularly with regard to maximising our understanding of astrophysical systems, geophysical regimes etc. In magnetic materials processing, rotation can also be utilized to achieve desired effects in the final products. Several studies of hydromagnetic convection in porous media with Hall effects have been reported. Takhar and Ram[29] have studied Hall current effects on natural MHD convection via a Darcian porous medium. Takhar et al.[30] have extended this model to study the supplementary effects of constant heat flux and heat generation combined with Hall currents and hydromagnetic drag on Darcian MHD convection in porous media. Again, a Darcian model was implemented in this analysis. We note that such a model is generally valid for Reynolds number up to 10, for which the flows are in the “creeping” regime i.e. the flow is viscous-dominated. In the present model, we study Hall/ionslip current, rotational, porous media drag force and temporal effects on the rotating magnetohydrodynamic heat transfer from a plate. Analytical solutions are obtained. Such a study has to the authors’ knowledge not appeared thus far in the literature.

2. MATHEMATICAL MODEL

We consider the unsteady hydromagnetic natural convection flow of a viscous, incompressible, partially-ionized fluid flowing adjacent to an impulsively started infinitely long vertical plate surface in an $X, Y, Z$ coordinate system embedded in a non-Darcy saturated porous medium. The plate surface
The equation of conservation of electrical charge:

Will be significant, constituting the ionslip effect. From

are the same. At τ = 0, the fluid and plate temperature are the same. At τ > 0, an impulsive motion is imposed on the plate in the X-Y plane so that the plate begins to move with velocity \( U_0 \) and experiences an instantaneous temperature rise to \( T_w \) and this is held constant. The magnetic Reynolds number is small for the partially-ionized fluid so that magnetic induction effects can be ignored. However, relative motion of the particles in the fluid can occur. As such, an electric current density, \( J \), is required to represent the relative motion of charged particles. Considering only the electromagnetic forces on these particles, we can utilize the generalized Ohm law. With a magnetic field, \( H \), applied normal to the electrical field \( E \), an electromagnetic force is generated normal to both \( E \) and \( H \). Such a force causes charged particles to migrate perpendicularly to both \( E \) and \( H \). Consequently, a component of electrical current density exists perpendicular to both \( E \) and \( H \), and this constitutes the Hall current. For a strong magnetic field \( H [H_x, H_y, H_z] \) the diffusion velocity of the ions will be significant, constituting the ionslip effect. From the equation of conservation of electrical charge:

\[
\nabla \cdot J = 0
\]

where \( J = (J_x, J_y, J_z) \). Since the plate is not composed of electrically-conducting material, electrical charge at the surface of the plate is constant and zero i.e. \( J_z \to 0 \). Consequently we can assume that \( J_z = 0 \) throughout the saturated porous medium. The non-zero electrical current density components \( J_x \) and \( J_y \) can now be obtained from the generalized Ohm law, following Sherman and Sutton[3] as:

\[
J_x = \left[ a (E_x + B_y v ) \right] \frac{(1 + \beta_x \beta_y)}{[(1 + \beta_x \beta_y)^2 + (\beta_y)^2]} \sigma \quad (2.2a)
\]

\[
J_y = \left[ a (E_y - B_x v ) \right] \frac{(1 + \beta_x \beta_y)}{[(1 + \beta_x \beta_y)^2 + (\beta_y)^2]} \sigma \quad (2.2b)
\]

where

\[
\beta_x = \frac{\beta_x}{[(1 + \beta_x \beta_y)^2 + (\beta_y)^2]}, \quad B_y = H_2 \quad \text{i.e.} \quad H_x = H_y = 0
\]

since there is no magnetic field in the X- and Y-directions, \( E_x \) is the X-direction electrical field, \( E_y \) is the Y-direction electrical field, \( \beta_x \) is the Hall parameter \( \left[ = \omega_e \chi_e \right], \beta_y \) is the ionslip parameter \( \left[ = \omega_i \chi_i \right] \), \( \omega_e \) and \( \omega_i \) are the electron and ion frequencies, \( \chi_e \) and \( \chi_i \) are the electron and ion collision frequencies and \( \sigma \) denotes the electrical conductivity of the partially-ionized fluid. We implement a Darcian drag force model defining the pressure gradient across the porous medium as:

\[
\nabla p = -aU
\]

where \( U \) denotes velocity, \( \nabla p \) is pressure gradient, \( a \) is a constant defined by \( a = \mu/k \) and \( \mu \) is the dynamic viscosity of the partially-ionised fluid, \( K \) is permeability [hydraulic conductivity] of the porous medium. We assume that the density of the partially-ionised fluid can be taken as constant i.e. the flow is incompressible. In addition, we implement the Boussinesq approximation which implies that all thermodynamic quantities of the fluid-saturated medium are constant, except for the buoyancy term, which is retained in the momentum conservation equation. We have assumed that the porous medium is homogenous and isotropic so that only a single permeability is needed to simulate hydraulic conductivity. Under these physical conditions, the flow regime in an [X, Y, Z] coordinate system can be represented by the following equations, neglecting convective acceleration terms \( \{ U \frac{\partial U}{\partial X} etc \} \), viz:

### X-direction Momentum Conservation

\[
\frac{\partial U}{\partial \tau} = -2\Omega V + v \frac{\partial^2 U}{\partial Z^2} + g \beta \{ T - T_w \} + \frac{1}{\rho} J_z B_y - v \frac{U}{K} \quad (2.4)
\]

### Y-direction Momentum Conservation

\[
\frac{\partial V}{\partial \tau} = 2\Omega U + v \frac{\partial^2 U}{\partial Z^2} - \frac{1}{\rho} J_x B_x - v \frac{V}{K} \quad (2.5)
\]

### Energy (Heat) Conservation

\[
\frac{\partial T}{\partial \tau} = \frac{k}{c_p} \frac{\partial^2 T}{\partial Z^2} \quad (2.6)
\]

where \( U \) and \( V \) are velocity components in the \( X \) and \( Y \) directions, \( \tau \) is time, \( v \) is kinematic viscosity of the partially-ionized fluid, \( g \) is acceleration due to gravity, \( \beta \) is the coefficient of volume expansion, \( \rho \) is density of the partially-ionized fluid, \( \kappa \) is thermal conductivity of the fluid-saturated porous medium, \( c_p \) is specific heat capacity of the partially-ionized fluid under isobaric conditions, \( T \) and \( T_\infty \) denote the boundary layer and free stream temperatures respectively, and \( J_x \) and \( J_y \) are components of electrical current density.

The physical model is illustrated below in Figure 1: In equations (2.4) and (2.5) the final terms on the right hand side are the X-direction Darcian drag force and the Y-direction Darcian drag force. The current model is valid for a short period after the impulsive motion commences and a temperature escalation takes place at the plate surface. The corresponding initial conditions and boundary conditions for the flow regime are as follows:

At \( \tau \leq 0, Z = 0 \):

\[
U = 0, \quad V = 0, \quad T = T_w \quad (2.7a)
\]

At \( \tau > 0, Z = 0 \):

\[
U = U_*, \quad V = 0, \quad T = T_w \quad (2.7b)
\]

At \( \tau > 0, Z \to \infty \):

\[
U \to 0, \quad V \to 0, \quad T \to T_\infty \quad (2.7c)
\]
In order to facilitate a solution to the coupled partial differential equations (2.4), (2.5) and (2.6), we introduce the following transformations, following Ram and Takhar [22]:

\[ t = \frac{x^2}{v} \]  
\[ z = \frac{ZU_o}{v} \]  
\[ u = \frac{U}{U_o} \]  
\[ v = \frac{V}{U_o} \]  
\[ \theta = \frac{T - T_o}{T_w - T_o} \]  
\[ \text{Pr} = \frac{\mu c_p}{k} \]  
\[ Gr = \frac{\nu z \beta_e (T_w - T_o)}{U_o^3} \]  
\[ Nm^2 = \frac{c_B \nu y}{\rho U_o^2} \]  
\[ Er = \frac{\Omega v}{U_o^3} \]  
\[ E_x = \frac{E_x}{B U_o} \]  
\[ E_y = \frac{E_y}{B U_o} \]  
\[ J_x = \frac{J_x}{c_B U_o} \]  
\[ J_y = \frac{J_y}{c_B U_o} \]  
\[ Da = \frac{k}{L^2} \]

where \( L \) denotes a characteristic length, \( Nm \) is the hydromagnetic parameter, \( \text{Pr} \) is Prandtl number, \( Gr \) is Grashof number, \( Er \) is the rotational parameter, \( \theta \) is dimensionless temperature, \( t \) is dimensionless time, \( z \) is dimensionless \( z \)-coordinate, \( u \) is dimensionless \( x \) direction velocity, \( v \) is dimensionless \( y \) direction velocity, \( E_x \) and \( E_y \) are dimensionless electrical field components in the transformed coordinate \((x,y)\) directions respectively, \( J_x \) and \( J_y \) are dimensionless electrical current densities in the transformed coordinate directions and \( Da \) is Darcy [porous bulk matrix resistance] number. Our transport equations now reduce to the following simplified pair of coupled partial differential equations, where \( q \) designates the sum of the real (i.e. primary velocity) and imaginary (i.e. secondary velocity) components = \( u + i v \), viz:

**Momentum Conservation**

\[
\frac{\partial q}{\partial t} + \frac{\partial q}{\partial z} + Gr \theta - M_1 q - H_o^2 \cdot \beta \left[ r - \frac{1}{DaRe^2}q \right] = 0
\]  

**Energy Conservation**

\[
\text{Pr} \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ \frac{q}{c_B} \right] - \frac{\partial}{\partial z} \left[ \frac{\nu z \beta_e (T_w - T_o)}{U_o^3} \right] \]

and here \( M_1 = 2Er + iH_o^2 \), \( q = u + iv \), \( E = E_x + iE_y \), \( H_o^2 = Nm^2 \cdot \alpha_e \) and \( \alpha_e = \alpha + i\beta \). The complex conjugate of \( q \) is denoted by the \( \bar{q} \) notation. The corresponding transformed boundary conditions now become:

At \( t \leq 0: \) \( q(z,t) = 0; \theta(z,t) = 0 \)  
(2.24a)

At \( t > 0: \) \( q(0, t) = 1, \theta(0, t) = 1 \)  
(2.24b)

At \( t > 0, \) \( q(\infty, t) = 0, \theta(\infty, t) = 0 \)  
(2.24c)

We note again that:

\[
\alpha = \frac{(1 + \beta_1 \beta_2)}{[(1 + \beta \beta_2)^2 + (\beta_2)^2]} \]  
(2.25)

\[
\beta = \frac{\beta_2}{[(1 + \beta \beta_2)^2 + (\beta_2)^2]} \]  
(2.26)

where \( \beta_1 \) is the Hall current parameter and \( \beta_2 \) is the ionslip current parameter.

### 3. SPECIAL CASES OF THE FLOW MODEL

A number of special cases can be derived from the full transformed momentum equation (24), which we shall now discuss:

**CASE I: Rotating MHD Shortcircuit Free Convection Flow in a Darcian Fluid-Saturated Porous Medium**

As \( E \rightarrow 0 \), electrical field effects vanish and the momentum equation (2.22) reduces to:

\[
\frac{\partial q}{\partial t} + \frac{\partial q}{\partial z} + Gr \theta - M_1 q - \frac{1}{DaRe^2}q = 0
\]  
(3.1)

**CASE II: Forced MHD Convection Flow in a Darcian Porous Medium with Hall and Ionslip Current Effects**
As $Gr \to 0$, buoyancy forces vanish and the general momentum equation (2.22) becomes decoupled from the energy equation (2.23), with (2.22) reducing to:

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} - M_q q - H_o^2 i E - \left[ \frac{1}{Da Re^2} \right] q$$

(3.2)

**CASE III: Forced Convection Short Circuit Flow in a Darcian Porous Medium**

In this case, again we set $E \to 0$, as well as $Gr \to 0$, which yields the following version of the transformed momentum equation (2.22), viz:

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} - M_q q - \left[ \frac{1}{Da Re^2} \right] q$$

(3.3)

**CASE IV: Free Convection MHD Flow with Hall and Ionslip Current Effects in a Purely Fluid Medium**

Setting $Da \to 0$, the medium fibers vanish i.e. the medium attains infinite permeability and becomes purely fluid. The Darcian drag force therefore vanishes and the generalized momentum equation (2.22) takes the form given by Ram and Takhar [22]:

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} + Gr \theta - M_q q - H_o^2 i E$$

(3.4)

**CASE V: Hydrodynamic Forced Convection in a Non-Rotating Darcian Porous Medium**

Setting $E \to 0$, $Gr \to 0$, and $M_1 = 2Er + iH_o^2 \to 0$ the electrical and magnetic field effect sand also rotational effects vanish so that the flow defines only hydrodynamic forced convection in a non-rotating Darcian porous regime. In this case equation (2.22) reduces to:

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2}$$

(3.5)

In all the above cases the energy (heat) equation is unaffected. We also note that for cases where $Gr < 0$, the plate surface is heated by free convection currents i.e. the plate draws heat from the fluid-saturated porous medium. Conversely, for $Gr > 0$, plate-cooling occurs. Backflow takes place in the vicinity of the edge of the boundary layer in both the primary velocity ($u$) and secondary velocity ($v$) profile, for $Gr < 0$. For $Gr > 0$, there will therefore be no backflow in the region of the boundary layer edge. Further discussion is provided in section 5.

### 4. ANALYTICAL SOLUTIONS USING LAPLACE TRANSFORM TECHNIQUE

Numerical solutions to the governing transformed equations (2.22) and (2.23) subject to boundary conditions 2.24(a,b,c) have been obtained. The generalized velocity and temperature distributions then take the form:

$$q(z,t) = \frac{1}{2} \left[ 1 - \frac{B_1}{Da} \right] e^{-zK_1} \text{erfc} \left( \frac{z - 2K_1 t}{2\sqrt{t}} \right) +$$

$$e^{2K_1} \text{erfc} \left( \frac{z + 2K_1 t}{2\sqrt{t}} \right)$$

(4.1)

$$\theta(z,t) = \left( \frac{Pr}{2} \right)^{\frac{1}{2}}$$

(4.2)

where:

$$K_1 = \left[ M_1 + \frac{1}{Da Re^2} \right]^{\frac{1}{2}}$$

(4.3a)

$$B_1 = \frac{Gr}{Pr - 1}$$

(4.3b)

$$D_1 = \frac{K_1^2}{Pr - 1}$$

(4.3c)

$$K_2 = C_s + iD_1$$

(4.3d)

$$C_s = \frac{1}{\sqrt{2\pi}} \left( S_1^2 + S_2^2 \right)^{1/2}$$

(4.3e)

$$S_1 = Nm^2 \alpha + \frac{1}{Da Re^2}$$

(4.3f)

$$S_2 = 2Er + Nm^2 \beta$$

(4.3g)

Clearly we may extract several special cases for the velocity field, $q(z,t)$.

**CASE A: Rotating MHD Shortcircuit Free Convection Flow in a Darcian Fluid-Saturated Porous Medium**

As $E \to 0$, electrical field effects vanish i.e. we obtain the “short circuit” model for which:

$$q(z,t) = \frac{1}{2} \left[ 1 - \frac{B_1}{Da} \right] e^{-zK_1} \text{erfc} \left( \frac{z - 2K_1 t}{2\sqrt{t}} \right) +$$

$$e^{2K_1} \text{erfc} \left( \frac{z + 2K_1 t}{2\sqrt{t}} \right)$$

(4.4)

$$\frac{B_1}{Da} \text{erfc} \left( \frac{z}{2\sqrt{t}} \right) + \frac{z}{4\sqrt{\pi}} \frac{B_1}{Da} \sqrt{2 - 1}$$

$$\left[ e^{z^2/2} - \text{erfc} \left( \frac{z}{2\sqrt{t}} \right) \right]$$

**CASE B: Forced Convection Short Circuit Flow in a Darcian Porous Medium**
With \( \epsilon \to 0 \), and \( \text{Gr} \to 0 \), then in (4.3b), 
\[
B_i = \left[ \frac{\text{Gr}}{\text{Pr} - 1} \right] \to 0,
\]
so that the velocity solution further reduces from (4.4) to:
\[
q(Z,t) = \frac{1}{2} e^{-2Kt} \text{erfc} \left[ \frac{z - 2Kt}{2\sqrt{t}} \right] + \nonumber
e^{2Kt} \text{erfc} \left[ \frac{z + 2Kt}{2\sqrt{t}} \right]
\]
(4.5)

**CASE C: Hydrodynamic Forced Convection in a non-Rotating Purely Fluid Regime**

Setting \( \epsilon \to 0 \), \( \text{Gr} \to 0 \), and \( M_1 = 2\epsilon \text{Re} + iH_0 \to 0 \) the electrical and magnetic field effects and also rotational effects vanish. With an infinite permeability, \( \text{Da} \to \infty \). Now with \( M_1 \to 0 \) then in (4.3a):
\[
K_i \left[ M_1 + \frac{1}{\text{DaRe}^2} \right]^2 \to 0
\]
(4.6)
and the velocity can be shown to take the elementary form:
\[
q(Z,t) = \text{erfc} \left[ \frac{z}{2\sqrt{t}} \right]
\]
(4.7)

**5. RESULTS AND DISCUSSION**

Extensive computations have been performed to examine the influence of \( \text{Da} \) (Darcy number), \( \epsilon \epsilon \) (rotational parameter), \( \beta_i \) (Hall current parameter), \( \beta_i \) (ionslip parameter), \( \text{Gr} \) (Grashof number i.e. free convection parameter) and \( \text{Nm} \) (hydromagnetic number) on primary and secondary velocity fields with spatial coordinate \( z \), at a fixed time \( t = 0.2 \). Additionally the influence of \( \text{Prandtl number} \) \( \text{Pr} \) on non-dimensional temperature \( \theta \) evolution in the regime has been computed at fixed dimensionless time \( t \) and also the variation of temperature \( \theta \) with time variable \( t \). Selected results are presented in figures 2 to 18. Recently numerical methods have been employed by the authors to study Hall current effects\(^{[31,32]} \) in hydromagnetic flows. The present analytical solutions therefore further serve to benchmark subsequent computational studies. Three special cases are considered. In figures 11 and 12, ionslip is neglected \( (\beta_i = 0) \). In figures 15 to 18 porous media drag is neglected (i.e. infinite permeability considered). Finally in figures 17 and 18, both ionslip and porous media effects are ignored. Otherwise all the graphs correspond to the general flow case with Hall and ionslip currents present in free convection through a rotating porous regime.

In figure 2 the variation of secondary velocity \( w \) with spatial coordinate \( z \) at \( t = 0.2 \) is illustrated for a range of Darcy numbers \( \text{Da} \). We note that in the present study a highly permeable regime is considered so that very high \( \text{Da} \) values are employed. An increase
in Da from 0.1 to 1 and 10 clearly increases the magnitude of the secondary velocity i.e. accelerates the secondary flow from the plate. This is particularly amplified at z~ 1 i.e. at a short distance from the plate surface. In the momentum equation (2.22), the porous medium drag force, $-\frac{1}{Da}Re q$, (in which $q = u + iv$, where $u$ and $v$ are the primary and secondary velocity components) is inversely proportional to the Darcy number for a fixed value of Reynolds number. The latter has been prescribed as 5.0 (valid for Darcian flows) and therefore an increase in Da will clearly reduce the magnitude of this porous drag contribution. Similarly using a physical argument, increasing Da will imply progressively less and less resistance from the solid material in the regime (i.e. porous medium fibers) to the flow. As such a rise in Da will increase both primary and secondary velocities in the regime. We note that in all computations (unless otherwise indicated), Gr = 5.0 i.e. Gr > 0. This implies that the plate is cooled by free convection currents.

In figures 3 and 4 we illustrate the response of primary and secondary velocity components to the variation in rotational parameter, $Er (= \frac{\Omega r}{U_o^2})$. This parameter defines the relative influence of Coriolis (fictitious rotational body force) to the inertial force. An increase in Er clearly implies an increase in $\Omega$ which corresponds to an escalation in the angular velocity of rotation the plate-fluid saturated porous medium system. For a non-rotating system, $Er = 0$. We observe that there is a distinct increase in primary velocity, $u$ as $Er$ increases from 0 through 0.5 to the maximum value of 1. Increasing rotational effects therefore accelerate the primary flow, but only in a weak fashion. Values however remain consistently positive indicating that there is no flow reversal in the primary flow direction i.e. back flow is absent at the edge of the boundary layer. Conversely in figure 4 there is substantial decrease in the secondary velocity, $v$, with an increase in $Er$. As such the secondary flow is continuously decelerated. All values are negative indicating that for all values of $Er$ (including the stationary plate scenario, $Er = 0$), there is strong backflow. The maximum backflow arises close to the plate at $z \sim 1$. Further from the plate backflow is reduced although never eliminated.

In figures 5 and 6 the response of the primary and secondary flow fields to various Hall current parameter values, $\beta_e$ are presented. Primary flow is seen to be accelerated with an increase in $\beta_e$ from 0 through 0.2, 0.4, 0.6 to 1. In this latter case a velocity overshoot is identified at $z \sim 1$. However with a further increase in Hall current effect, the primary velocity is infact decreased considerably with the onset of flow reversal close to the plate surface; this back flow for $\beta_e = 5$ is sustained with further separation from the plate at large distances external to the boundary-layer regime. As such Hall current is only beneficial to the primary flow regime for $\beta \leq 1$; thereafter increasing the Hall effect acts to impede the primary flow. Such a feature is of clear significance in the operation of MHD energy systems in which larger Hall current values may be utilized to effectively control primary flow dynamics. We note that the Hall current parameter is simulated in our model via the expression $H_e^2 = N m^2 \alpha_o$ in which $\alpha_o = \alpha + i \beta$ with

$$\alpha = \frac{(1 + \beta_2 \beta_e)}{[(1 + \beta_2 \beta_e)^2 + (\beta_e)^2]} \quad \text{and} \quad \beta = \frac{\beta_e}{[(1 + \beta_2 \beta_e)^2 + (\beta_e)^2]}$$

For the case of no Hall current present, $\beta_e \rightarrow 0$ and the expressions contract to: $\alpha = 1$ and $\beta = 0$. In other words, ionslip effects ($\beta_i$) are also negated for the vanishing Hall current case. For this case a sharp descent in primary flow occurs from the plate surface to relatively low values throughout the remainder of the domain. For $\beta_e = 1$ the primary velocities attain significantly higher values and for $\beta_e \geq 0$, there is no presence of flow reversal anywhere in the regime. A substantially different response is observed for the secondary flow ($v$) as shown in figure 6. For $\beta_e = 0$, no response is observed for the secondary flow; with increasing positive values of $\beta_e$ through 0.2, 0.4, 0.6 to 1, a strong decrease in secondary velocity ($v$) is observed i.e. increasingly stronger back flow arises throughout the domain normal to the plate surface (along the $z$-coordinate). Maximum backflow accompanies the case of $\beta_e = 1$, the exact opposite to the primary flow response (where maximum acceleration is associated with $\beta_e = 1$). Conversely with subsequent increase in $\beta_e$ to 5 a marked acceleration arises in the secondary flow; an approximate symmetry of v-profiles is apparent for the $\beta_e = 1$ and $\beta_e = 5$ cases. The Hall current induces a strong secondary flow i.e. cross flow and for $\beta_e > 1$ this aids the secondary flow development in the regime. In all cases the secondary flow is weakened at large distances from the plate surface. For $\beta_e \leq 1$ the secondary flow is reversed throughout the regime. Maximum velocity in the secondary flow arises at the velocity overshoot location i.e. at $z \sim 1$ for $\beta_e = 5$. These results therefore indicate that in for example MHD energy generators, to achieve a strong, positive secondary (cross-) flow, very high Hall currents must be present.

In figures 7 and 8 the distributions of primary and secondary velocity for various ion slip parameter values, $\beta_i$ are presented. Primary velocity is seen to be increased slightly with increase in $\beta_i$ from 0 (no ionslip) through 0.4 and 1; maximum effects of ionslip on the profiles arise at $z \sim 1$ i.e. close to the plate surface. A weak oscillatory pattern also arises near the location $z \sim 1$ with all profiles falling to the lowest value far from the plate. There is however no presence of backflow anywhere in the porous regime. Effectively primary flow is accelerated weakly with an escalation in ionslip parameter. For the case of the secondary flow, for all values of the ionslip parameter
(β) significant flow reversal arises throughout the porous medium. The strongest backflow is associated with an absence of ionslip currents (β = 0). With an increase in β, the secondary velocity is slightly increased. The presence of ionslip currents therefore exerts a much weaker influence on the secondary flow compared with Hall currents (Figure 6). These trends agree with the non-porous results of, for example, Ram and Takhar[22] and Ram et al[23].

In Figures 9 and 10 the effects of thermal Grashof number on primary and secondary velocity profiles are depicted, again for the case of a highly porous medium (Da = 1.0). In thermal MHD energy systems, three cases are of interest: pure forced convection (Gr = 0), plate cooling with free convection currents (Gr > 0) and plate heating with free convection currents (Gr < 0). For the forced convection case (Gr = 0) we observe that primary velocity (Figure 9) descends rapidly from the peak at the plate surface (z = 0) and a slight backflow is apparent close to the plate surface; thereafter negligible velocities are recorded i.e. the flow is stagnated further from the plate. Secondary velocity is completely eliminated for the forced convection case. For Gr > 0 (i.e. 5, 10, 20) with increasing thermal buoyancy force in the saturated porous regime, a distinct acceleration in primary velocity is caused. For the strongest buoyancy case (Gr = 20) a primary velocity overshoot arises in the vicinity of z ≈ 1. Further from this point the primary velocity profile decays smoothly into the far field regime, with no back flow induced anywhere in the regime. For Gr < 0 however strong flow reversal is generated; increasingly negative Grashof numbers (-5, -10, -20) result in progressively stronger backflow. A negative primary velocity overshoot accompanies the case of strongest plate heating by free convection currents (Gr = -20). The reverse pattern is witnessed in the case of the secondary flow (Figure 10). Increasing positive Gr values (plate cooling) consistently decelerate the flow and induce substantial back flow. For increasingly negative Gr values (plate heating), the secondary flow is conversely accelerated i.e. positive secondary velocity profiles are produced with rising values of Gr with the complete elimination of backflow in the porous regime.

The effects of the magnetohydrodynamic body force parameter, Nm, on u- and v-distributions is provided in Figures 11 and 12. In both these figures we have neglected ion slip current effects (β = 0). Nm² = \frac{\partial B_y}{\partial z} and as such an increase in transverse magnetic field, B_y, will generate a direct increase in the Nm value. Hence stronger magnetic field effects are associated with larger Nm values. For the case of Nm = 1 the magnetohydrodynamic and viscous forces in the regime will have approximately the same order of magnitude. An increase in Nm from 5 through 7, 9 and 12, clearly increases the magnitude of the Lorentz hydromagnetic drag force which acts perpendicularly to the applied magnetic field and thereby serves to decelerate the primary flow i.e. a strong reduction in u-profile values is caused, as shown in Figure 11. Backflow however is never generated in the primary flow. On the other hand, the same increase in Nm serves to reduce back flow in the secondary flow i.e. decreases magnitudes of the v-velocity profiles. In all cases however, due to the presence of Hall currents (β = 0.5) which is associated with secondary flows (cross-flows), large values are generated for the secondary flow velocity. Increasing magnetic field therefore decreases backflow in the secondary flow.

In Figures 13 and 14 the primary and secondary velocity profiles are presented again for Nm effects but in the presence of ionslip currents (β = 0.2). Very little difference is observed between these profiles and those in Figures 11 and 12, confirming the relatively weak influence of ion slip currents on the regime. In other words the dominant magnetofluid effects are associated with the Lorentzian magnetic drag force and the Hall currents, as confirmed by Sherman and Sutton[22] and Bhat and Mittal[28,8,10].

Figures 15 and 16 illustrate the influence of the rotational parameter, Er, on u- and v-distributions for the purely fluid case i.e. in the absence of Darcian porous drag effects (Da = 0). The profiles are very similar to those in Figures 3 and 4 for a highly porous medium (Da = 1). As such the similarity indicates that the further reduction in drag force from the case of a highly porous to a purely fluid medium will have minimal influence on the velocity evolution in the regime. However a reduction in Da to very small values, as discussed earlier, will serve to increase radically the porous medium impedance and this be exploited in regulating the flow in the porous regime.

Finally in Figure 17 the distribution of temperature, θ, throughout the regime is illustrated for different Prandtl numbers, Pr. In all cases, values correspond to liquid metals which possess very high thermal conductivities i.e. very low Pr values. Smaller Pr values are associated with thicker boundary layers in the vicinity of the plate. Heat diffuses in lower Pr fluids faster from the plate than for higher Pr fluids. As such the temperatures in the medium (which is purely fluid for this case i.e. Da → 0) are decreased continuously from the wall into the medium with an increase in Pr.

In Figure 18, we also observe that with an increase in time (t), temperatures are however increased consistently since for Gr > 0 the plate is cooled by free convection currents, which serves to transfer thermal energy to the regime and warms it.

7. CONCLUSIONS

Unsteady magnetohydrodynamic flow from a rotating plate with thermal convection in a porous regime in
the presence of Hall and ionslip current effects has been studied theoretically. The non-dimensional conservation equations have been solved under appropriate boundary conditions analytically. The results have generally been computed for the case of liquid metal (low Prandtl number) at a fixed time to simulate possible regimes in a hybrid MHD porous medium energy generator system. Our results indicate that:

a) An increase in Darcy number \( (Da) \) (corresponding to a rise in permeability of the porous regime) increases both primary \( (u) \) and secondary \( (v) \) velocities in the regime.

b) An increase in the rotation parameter \( (Er) \) acts to accelerate the primary flow \( (u) \) albeit weakly with no back flow, but acts to significantly decrease the secondary velocity \( (v) \).

c) Primary flow \( (u) \) is accelerated with an increase in Hall current parameter \( (\beta_e) \) up to \( \beta_e = 1 \); with subsequent increase in \( \beta_e \) the primary velocity is reduced markedly accompanied with strong back flow. An increase in \( \beta_e \) effectively decreases secondary flow velocity \( (v) \) up to \( \beta_e = 1 \) with strongest flow reversal associated with \( \beta_e = 1 \); with further rise in the Hall current parameter \( (\beta_e) \) to 5 however secondary flow is boosted i.e. accelerated.

d) Primary flow is accelerated weakly with an increase in ionslip parameter \( (\beta_i) \) and secondary velocity \( (v) \) is also slightly enhanced with strong backflow present in the latter case.

e) For \( Gr > 0 \) (i.e plate colling with free convection currents) primary velocity is greatly increased with the reverse response for \( Gr < 0 \) which induces strong backflow. For increasingly negative \( Gr \) values (plate heating), the secondary flow is conversely accelerated with the reverse behaviour apparent for positive \( Gr \) values (plate cooling).

f) An increase in magnetohydrodynamic drag force parameter \( , Nm \), decelerates the primary flow but serves to reduce back flow in the secondary flow.

g) Ion slip currents are found to have a much weaker effect on the fluid dynamics of the system comapred with the magnetic drag force and the Hall current effect.

h) Temperatures in the medium are decreased consistently with an increase in Prandtl number but are increased with greater time elapse \( (t) \) due to plate cooling by free convection currents \( (Gr > 0) \).

The current study has examined the case of Newtonian fluid. Future studies will address non-Newtonian effects including couple stress (polar) fluids, micropolar fluids and liquid crystals. The results of these studies will be communicated imminently.

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Figure 5: Primary velocity \((u)\) versus coordinate normal to plate surface \((z)\) for various \(\beta_e\) values.

Figure 6: Secondary velocity \((v)\) versus coordinate normal to plate surface \((z)\) for various \(\beta_i\) values.

Figure 7: Primary velocity \((u)\) versus coordinate normal to plate surface \((z)\) for various \(\beta_i\) values.

Figure 8: Secondary velocity \((v)\) versus coordinate normal to plate surface \((z)\) for various \(\beta_e\) values.

Figure 9: Primary velocity \((u)\) versus coordinate normal to plate surface \((z)\) for various \(Gr\) values.

Figure 10: Secondary velocity \((v)\) versus coordinate normal to plate surface \((z)\) for various \(Gr\) values.
Figure 11: Primary velocity ($u$) versus coordinate normal to plate surface ($z$) for various $Nm$ values in the absence of ionslip currents ($\beta_i = 0$)

Figure 12: Secondary velocity ($v$) versus coordinate normal to plate surface ($z$) for various $Nm$ values in the absence of ionslip currents ($\beta_i = 0$)

Figure 13: Primary velocity ($u$) versus coordinate normal to plate surface ($z$) for various $Nm$ values in the presence of ionslip currents ($\beta_i = 0.2$)

Figure 14: Secondary velocity ($v$) versus coordinate normal to plate surface ($z$) for various $Nm$ values in the presence of ionslip currents ($\beta_i = 0.2$)

Figure 15: Primary velocity ($u$) versus coordinate normal to plate surface ($z$) for various $Er$ values for a purely fluid regime ($Da = \infty$)

Figure 16: Secondary velocity ($v$) versus coordinate normal to plate surface ($z$) for various $Er$ values for a purely fluid regime ($Da = \infty$)
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