

# INFLUENCE OF CHEMICAL REACTION ON TRANSIENT MHD FREE CONVECTIVE FLOW OVER A VERTICAL PLATE IN SLIP-FLOW REGIME

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يقدم البحث دراسة عن أثر الهيدروديناميكية المغناطيسية (MHD) والتفاعلات الكيميائية على ما يلي: الجريان لمائع غير مستقر مع الزمن وعلى كل من انتقال الحرارة والمادة تحت ظروف جريان مائع لزج، غير قابل للانضغاط، يتسم بالقدرة على التوصيل الكهربائي. الجريان يتم فوق سطح شبه لانتهائي عمودي قابل للتنفيذ. تم اعتماد الجريان المنزلق (slip-flow). السطح العمودي تم تعريضه لسرعة شفط عمودية على اتجاه الجريان. لقد تم حل المعادلات المترابطة غير المستقرة مع الزمن وغير الخطية باستخدام طريقة قلقة لسلسلة متوسعة حول معامل قلقة صغير. النتائج تم تطبيقها على كل من الهواء والماء تحت ظروف عدد براندتل 0.71 و 7، بالنتائج. تخلص الدراسة إلى أن التفاعل الكيماوي له أثر معاكس للسرعة وعلى توزيع التركيز، في حين أن معامل التخلخل يؤدي إلى زيادة السرعة غير المستقرة ومستوى الاحتكاك السطحي، في المقابل، يؤثر المجال المغناطيسي عكسياً على الجريان وعلى كل من معامل الاحتكاك ووجهه.

In the present study, an analysis is carried out to study the Magnetohydrodynamic and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conduction fluid over a semi-infinite vertical porous plate in a slip-flow regime. The porous plate is subjected to a transverse variable suction velocity. The transient, non-linear and coupled governing equations have been solved adopting a perturbative series expansion about a small parameter,  $\epsilon$ . The effects of governing parameters on the flow variables are discussed quantitatively with the aid of graphs for the flow field, concentration field and fluctuating parts of both the skin-friction and the rate of mass transfer. All numerical calculations are done with respect to air at 20°C ( $Pr = 0.71$ ) as well as water ( $Pr = 7$ ) in presence of diffusing chemical species has Schmidt number  $Sc = 0.22$ . Comparison between the results of the present analytical model with previously published works provides excellent agreement. It is found that, the chemical reaction parameter has a retarding effect on the velocity of flow field as well as concentration distributions, while rarefaction parameter leads to increase both the transient velocity and amplitude of skin-friction. However, the magnetic effect is found to decrease the flow field and both the amplitude as well as phase of skin-friction. During the course of discussion, it is found that chemical reaction, rarefaction parameter as well as magnetic field appreciably influence the fluid flow.

**Keywords:** Transient velocity, MHD, Mass Transfer, Chemical reaction and Rarefaction parameter.

## 1. INTRODUCTION

In many transport processes existing in nature and in industrial applications in which heat and mass transfer is a consequence of buoyancy effects caused by diffusing of heat and chemical species. The study of such processes is useful for improving a number of chemical technologies, such as polymer production, enhanced oil recovery, underground energy transport, manufacturing of ceramic and food processing. A clear understanding of the nature of interaction between

thermal and concentration buoyancies is necessary to control these processes. In nature, the presence of pure air or water is impossible. Some foreign mass may be present either naturally or mixed with the air or water. The effect of the presence of foreign mass on the free convection flow past a semi-infinite vertical plate was studied by Gebhart and Pera<sup>[1]</sup>. During a chemical reaction between two species, heat is also generated. In most of the cases of chemical reaction, the reaction rate depends on the concentration of species itself. A reaction is said to be first order if the rate of reaction is

directly proportional to concentration itself. The problems of steady and unsteady have combined heat and mass transfer by free convection along an infinite and semi-infinite vertical plate with and without chemical reaction have been studied extensively by different scholars<sup>[1-22]</sup>. Ganesan and Loganathan<sup>[2]</sup> presented numerical solutions of the transient natural convection flow of an incompressible viscous fluid past an impulsively stated semi-infinite isothermal plate with mass diffusion, taking into account a homogeneous chemical reaction of first order. Ghaly and Seddeek<sup>[3]</sup> analyzed the effect of variable viscosity; chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate. Muhaimin *et al.*<sup>[4]</sup> analyzed the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer past a porous shrinking sheet with suction.

Muthucumaraswamy and Ganesan<sup>[5]</sup> studied numerically the transient incompressible viscous fluid flow regime past a semi-infinite isothermal plate under the conditions of natural convection. Rahman and Mulolani<sup>[6]</sup> studied the laminar natural convection flow over a semi-infinite vertical plate at constant species concentration. They found that in the absence of chemical reaction, a similarity transform is possible, while when chemical reaction occurs, perturbation expansions about an additional similarity variable dependent on reaction rate must be employed. Muthucumaraswamy and Kulaivel<sup>[7]</sup> presented an analytical solution to the problem of flow past an impulsively started infinite vertical plate in the presence of uniform heat flux and variable mass diffusion, taking into account the homogeneous chemical reaction of first order. Hossain *et al.*<sup>[8]</sup> studied the influence of fluctuating surface temperature and concentration on natural convection flow from a vertical plate. Mansour *et al.*<sup>[9]</sup> investigated the effects of chemical reaction and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous media considering Soret and Dufour numbers.

Magnetic fields influence many natural and man-made flows. There are routinely used in industry to heat, pump, stir, and levitate liquid metals. There is the terrestrial magnetic field, which is maintained by fluid motion in the earth's core, the solar magnetic field that generates sunspots and solar flares, and the galactic field which influences the formation of stars. The flow problems of an electrically conducting fluid under the influence of magnetic field have attracted the interest of many authors in view of their applications to geophysics, astrophysics, engineering, and to the boundary layer control in the field of aerodynamics. On the other hand, in view of the increasing technical applications using Magnetohydrodynamics (MHD) effect, it is desirable to extend many of the available viscous hydrodynamic solutions to include the effects of magnetic field for those cases when the viscous

fluid is electrically conducting. Recently, The chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification have been presented by Kandasamy *et al.*<sup>[10]</sup>. The opposing buoyancy effects on simultaneous heat and mass transfer by natural convection in a fluid saturated porous medium investigated by Angirasa *et al.*<sup>[11]</sup>. Ahmed<sup>[12]</sup> investigates the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Later on, Ahmed<sup>[13]</sup> studied the effects of heat and mass transfer on the steady three-dimensional flow of a viscous incompressible fluid along a steadily moving porous vertical plate subjected to a transverse sinusoidal suction velocity. Also, the effects of heat and mass transfer on the unsteady free convective flow of a viscous incompressible fluid past an infinite vertical porous plate in presence of time dependent transverse suction velocity had been presented by Ahmed<sup>[14]</sup>. Perturbation analysis of unsteady Magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction were investigated by Pal and Talukda<sup>[15]</sup>.

The present analysis discussed here has many applications as suggested by Soundalgekar and Wavre<sup>[16, 17]</sup>. In many practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle at the surface has a finite tangential velocity; it "slips" along the surface. The flow regime is called the slip-flow regime and this effect cannot be neglected. Using these assumptions, Sharma and Chaudhary<sup>[18]</sup> discussed the free convection flow past a vertical plate in slip-flow regime and also discussed the free convection flow past a vertical plate in slip-flow regime and also discussed its various applications for engineering purpose. Also, Sharma<sup>[19]</sup> investigate the effect of periodic heat and mass transfer on the unsteady free convection flow past a vertical flat plate in slip-flow regime when suction velocity oscillates in time. Chaudhary and Jha<sup>[20]</sup> studied the effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime. Moreover, Al-Odat and Al-Azab<sup>[21]</sup> studied the influence of magnetic field on unsteady free convective heat and mass transfer flow along an impulsively started semi-infinite vertical plate taking into account a homogeneous chemical reaction of first order.

As a step towards the eventual development in the study of transient MHD free convective chemically reacting fluid in a slip flow regime, in the present investigation, it is proposed to obtain the analytical solution for the unsteady MHD free convection flow over a semi-infinite vertical porous plate including the effects of magnetic field, chemical reaction, rarefaction and the wall suction velocity. The unsteadiness is introduced in the flow field by the time dependent wall suction velocity. The solution of the

coupled non-linear partial differential equations governing free convection flow, heat and mass transfer has been obtained analytically using the perturbation technique. The fluids considered in this investigation are air ( $Pr = 0.71$ ) and water ( $Pr = 7$ ) in presence of Hydrogen ( $Sc = 0.22$ ).

## 2. MATHEMATICAL ANALYSIS

The transient MHD free convection flow of an electrically conducting, viscous incompressible fluid over a porous vertical infinite isothermal plate in slip-flow regime with chemical reaction of first order has been presented in Figure 1.

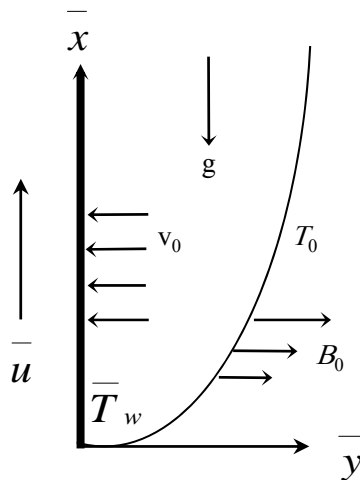


Figure 1 The physical configuration of the problem

The  $\bar{x}$ -axis is assumed to be along the plate and the  $\bar{y}$ -axis normal to the plate. Since the plate is considered infinite in  $\bar{x}$ -direction, hence all physical quantities will be independent of  $\bar{x}$ . Therefore, all the physical variables become functions of  $\bar{y}$  and  $\bar{t}$  only. The wall is maintained at constant temperature  $\bar{T}_w$  and concentration  $\bar{C}_w$  higher than the ambient temperature  $\bar{T}_\infty$  and concentration  $\bar{C}_\infty$  respectively. The viscous dissipation and the Joule heating effects are assumed to be negligible in the energy equation. Also, it is assumed that there is a homogeneous chemical reaction of first order with rate constant  $\bar{K}$  between the diffusing species and the fluid. A uniform magnetic field of magnitude  $B_0$  is applied normal to the plate. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible<sup>[22]</sup>. Also it is assumed that there is no applied voltage, so that the electric field is absent. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species, which are present, and hence the Soret and Dufour effects are negligible. Under the above assumptions as well as Boussinesq approximation, the equations of conservation of mass, momentum, energy and concentration governing the free convection

boundary layer flow over a vertical porous plate can be expressed as:

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (2.1)$$

$$\begin{aligned} \frac{\partial \bar{u}}{\partial \bar{t}} - v_0 (1 + \varepsilon A e^{i\bar{\omega}\bar{t}}) \frac{\partial \bar{u}}{\partial \bar{y}} &= g \beta (\bar{T} - \bar{T}_\infty) \\ + g \bar{\beta} (\bar{C} - \bar{C}_\infty) + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2}{\rho} \bar{u} \end{aligned} \quad (2.2)$$

$$\rho C_p \left[ \frac{\partial \bar{T}}{\partial \bar{t}} - v_0 (1 + \varepsilon A e^{i\bar{\omega}\bar{t}}) \frac{\partial \bar{T}}{\partial \bar{y}} \right] = \kappa \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \quad (2.3)$$

$$\begin{aligned} \frac{\partial \bar{C}}{\partial \bar{t}} - v_0 (1 + \varepsilon A e^{i\bar{\omega}\bar{t}}) \frac{\partial \bar{C}}{\partial \bar{y}} &= D \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} - \bar{K} (\bar{C} - \bar{C}_\infty) \end{aligned} \quad (2.4)$$

The corresponding boundary conditions of the problem are:

$$\left. \begin{aligned} \bar{u} &= L \left( \frac{\partial \bar{u}}{\partial \bar{y}} \right), \quad \bar{T} = \bar{T}_w + (\bar{T}_w - \bar{T}_\infty) e^{i\bar{\omega}\bar{t}}, \\ \bar{C} &= \bar{C}_w + (\bar{C}_w - \bar{C}_\infty) e^{i\bar{\omega}\bar{t}} \quad \text{at } \bar{y} = 0 \\ \bar{u} &\rightarrow 0, \quad \bar{T} \rightarrow \bar{T}_\infty, \quad \bar{C} \rightarrow \bar{C}_\infty \quad \text{at } \bar{y} \rightarrow \infty \end{aligned} \right\} \quad (2.5)$$

From the equation of continuity (2.1), it is clear that the suction velocity at the plate is either a constant or a function of time only. Hence, the suction velocity normal to the plate is assumed to be in the form:

$$\bar{v} = -v_0 (1 + \varepsilon A e^{i\bar{\omega}\bar{t}}) \quad (2.6)$$

We now introduce the following non-dimensional quantities into the equations (2.1) to (2.5):

$$y = \frac{v_0 \bar{y}}{\nu}, \quad u = \frac{\bar{u}}{v_0}, \quad t = \bar{t} v_0^2 / 4\nu, \quad \omega = 4\bar{\omega}\nu / v_0^2,$$

$$\nu = \mu / \rho, \quad Pr = \frac{\mu C_p}{\kappa},$$

$$\theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, \quad \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty},$$

$$Gr = \frac{\nu g \beta (\bar{T}_w - \bar{T}_\infty)}{v_0^3},$$

$$Gc = \frac{\nu g \bar{\beta} (\bar{C}_w - \bar{C}_\infty)}{v_0^3}, \quad M = \sigma B_0^2 \nu / \rho v_0^2,$$

$$Sc = \nu / D, \quad h = \frac{v_0 L}{\nu}, \quad K = \frac{\nu \bar{K}}{v_0^2}.$$

The governing equations (2.2) to (2.4) can be rewritten in the non-dimensional form as follows:

$$\frac{1}{4} \frac{\partial u}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr \theta + Gc\phi - Mu \tag{2.7}$$

$$\frac{1}{4} \frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \tag{2.8}$$

$$\frac{1}{4} \frac{\partial \phi}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - K\phi \tag{2.9}$$

The transformed boundary conditions are:

$$\left. \begin{aligned} u &= h \frac{\partial u}{\partial y}, \quad \theta = 1 + \varepsilon e^{i\omega t}, \\ \phi &= 1 + \varepsilon e^{i\omega t} \quad \text{at } y = 0 \\ u &\rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0, \quad \text{at } y \rightarrow \infty \end{aligned} \right\} \tag{2.10}$$

### 3. SOLUTION METHODOLOGY

The equations (2.7) to (2.9) are coupled, non-linear partial differential equations and these cannot be solved in closed form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. So this can be done, when the amplitude of oscillations ( $\varepsilon \ll 1$ ) is very small, we can assume the solutions of flow velocity  $u$ , temperature field  $\theta$  and concentration  $\phi$  in the neighbourhood of the plate as:

$$f(y, t) = f_0(y) + \varepsilon e^{i\omega t} f_1(y) + \dots, \tag{3.1}$$

where  $f$  stands for  $u, \theta$  or  $\phi$ .

Substituting (3.1) in (2.6) to (2.8), equating the coefficients of harmonic and non-harmonic terms, neglecting the coefficients of  $\varepsilon^2$ , we get:

$$\theta_0'' + Pr \theta_0' = 0, \tag{3.2}$$

$$\theta_1'' + Pr \theta_1' - i\omega Pr \theta_1 / 4 = -2A Pr \theta_0', \tag{3.3}$$

$$u_0'' + u_0' - M u_0 = -Gr \theta_0 - Gc \phi_0, \tag{3.4}$$

$$u_1'' + u_1' - (M + i\omega/4) u_1 = -Gr \theta_1, \tag{3.5}$$

$$\phi_0'' + Sc \phi_0' - k Sc \phi_0 = 0, \tag{3.6}$$

$$\phi_1'' + Sc \phi_1' - Sc(k + i\omega/4) \phi_1 = A Sc \phi_0', \tag{3.7}$$

where prime denotes differentiation with respect to  $y$ . The corresponding boundary conditions now are:

$$\left. \begin{aligned} u_0 &= h \left( \frac{\partial u_0}{\partial y} \right), \quad u_1 = h \left( \frac{\partial u_1}{\partial y} \right), \quad \theta_0 = 1, \\ \theta_1 &= 1, \quad \phi_0 = 1, \quad \phi_1 = 1, \quad \text{at } y = 0 \\ u_0 &\rightarrow 0, \quad u_1 \rightarrow 0, \quad \theta_0 \rightarrow 0, \quad \theta_1 \rightarrow 0, \\ \phi_0 &\rightarrow 0, \quad \phi_1 \rightarrow 0, \quad \text{at } y \rightarrow \infty \end{aligned} \right\} \tag{3.8}$$

The solutions of the equations (3.2) to (3.7) under the boundary the conditions (3.8) are:

$$\theta_0(y) = e^{-Pr y}, \tag{3.9}$$

$$\phi_0(y) = e^{-n y}, \tag{3.10}$$

$$\theta_1(y) = B_1 e^{-\lambda y} + B_2 e^{-Pr y}, \tag{3.11}$$

$$u_0(y) = B_5 e^{-q y} - B_3 e^{-Pr y} - B_4 e^{-n y}, \tag{3.12}$$

$$\phi_1(y) = B_7 e^{-\mu y} - B_6 e^{-n y}, \tag{3.13}$$

$$\begin{aligned} u_1(y) &= B_{15} e^{-\Psi y} - B_{10} e^{-\lambda y} - B_{11} e^{-\mu y} \\ &+ B_{12} e^{-q y} - B_{13} e^{-Pr y} - B_{14} e^{-n y} \end{aligned} \tag{3.14}$$

### 4. SKIN-FRICTION AND MASS TRANSFER

Due to a fluid motion, the dimensionless shearing stress on the surface of a body is known as skin-friction and is defined by the Newton's law of viscosity

$$\bar{\tau} = \mu \frac{\partial u}{\partial y} \tag{4.1}$$

With the help of (3.12), (3.14) and (4.1), the shearing stress component at the plate can be calculated in non-dimensional form as:

$$\begin{aligned} \tau &= \frac{\bar{\tau}}{\rho v_0^2} = \frac{\partial u}{\partial y} \Big|_{y=0} \\ &= \tau_m + \varepsilon |F| \cos(\omega t + \alpha), \end{aligned} \tag{4.2}$$

where

$$\begin{aligned} \tau_m \text{ (Mean skin-friction)} &= -qB_5 + Pr B_3 + n B_4, \\ F &= F_r + i F_i = \lambda B_{10} + \mu B_{11} - q B_{12} + Pr B_{13} \\ &+ n B_{14} - \Psi B_{15}, \quad \tan \alpha = F_i / F_r. \end{aligned}$$

The relation between species transfer by convection and the concentration boundary layer may be demonstrated by recognizing that the molar flux associated with species transfer by diffusion, according to Fick's law, it has the form

$$\bar{q}_m = -D \left( \frac{\partial \bar{C}}{\partial y} \right)_{y=0} \tag{4.3}$$

With the help of (3.10), (3.13) and (4.3), the non-dimensional mass flux at the plate  $y = 0$  can be calculated in terms of Sherwood number as

$$\begin{aligned} Sh &= \frac{\bar{q}_m}{D(\bar{C}_w - \bar{C}_\infty)} = -\frac{\partial \phi}{\partial y} \Big|_{y=0} \\ &= -n + \varepsilon |H| \cos(\omega t + \beta), \end{aligned} \tag{4.4}$$

where  $H = H_r + i H_i = n B_6 - \mu B_7, \tan \beta = H_i / H_r$ .

### 5. VALIDATION OF THE PRESENT WORK

The accuracy of the present work has been verified by comparing its certain results with those available in the literature. The unsteady results without magnetic fields as well as chemical reaction are compared with<sup>[19]</sup> through Table 1 and 2 as well as Figure 2. It can be seen from Table 1 and 2 and Fig. 2, that the agreement between the results is excellent. This has established confidence in the analytical results reported in this paper.

Table 1: Comparison of the phase ( $\tan\alpha$ ) of skin-friction with<sup>[19]</sup>:

$h = 0.4, Gr = 2 = Gc, Pr = 0.71, \omega = 2$			
Authors	A	Sc = 0.94	Sc = 0.60
Sharma <sup>[19]</sup>	0.2	-1.1822	-1.2314
	0.6	-0.777	-0.8313
	1.0	-0.5767	-0.6491
Present work	0.2	-1.2076	-1.2883
	0.6	-0.8105	-0.9174
	1.0	-0.5972	-0.7109

Table 2: Comparison of the phase ( $\tan\beta$ ) of the rate of heat transfer when Sc = 0.94 with<sup>[19]</sup>:

Authors	A	$\omega = 5.0$	$\omega = 10$
Sharma <sup>[19]</sup>	0.2	0.5083	0.6321
	0.6	0.3886	0.5406
	1.0	0.2925	0.4611
Present work	0.2	0.5516	0.6702
	0.6	0.4301	0.6071
	1.0	0.3274	0.5332

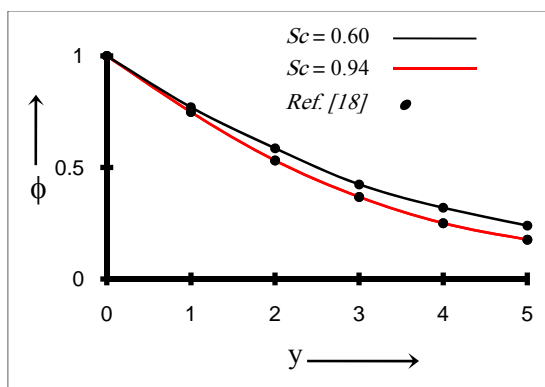


Figure 2 Comparison of dimensionless Concentration distributions of the present work and [18].

#### Particular cases:

(a) In the absence of magnetic field ( $M = 0$ ) as well as Chemical Reaction ( $k = 0$ ), the results of the

present paper are reduced to those obtained by Sharma<sup>[19]</sup>.

(b) In the absence of magnetic field ( $M = 0$ ), mass transfer (i.e. thermal Grashof number) as well as Chemical Reaction ( $k = 0$ ), the results of the present paper are reduced to those obtained by Sharma and Choudhury<sup>[18]</sup>.

(c) Without slip-flow regime (i.e. rarefaction rate  $h = 0$ ), magnetic field ( $M = 0$ ) as well as Chemical Reaction ( $k = 0$ ), the results of the present paper are reduced to those obtained by Soundalgekar and Wavre<sup>[16]</sup>.

(d) If we omit the rate of rarefaction parameter, the behaviour the concentration under the influence of chemical reaction is similar with the previously published works Al-Odat and Al-Azab<sup>[21]</sup>.

### 5. RESULTS AND DISCUSSION

It is very difficult to study the influence of all governing parameters involved in the present problem on the transient MHD flow with periodic heat and mass transfer in presence of chemical reaction of first order. Therefore, this study is focused on the effects of governing parameters on the transient velocity as well as on the concentration profiles. To have a physical feel of the problem we, exhibit results to show how the material parameters of the problem affect the velocity, and Concentration profiles. Here we restricted our discussion to the aiding of favourable case only, for fluids with Prandtl number  $Pr = 0.71$  which represent air at 20 °C at 1 atmosphere and for fluids  $Pr = 7$  which represent water. The value of thermal Grashof number  $Gr$  is taken to be positive, which corresponds to the cooling of the plate. The diffusing chemical species of most common interest in air has Schmidt number ( $Sc$ ) and is taken for Hydrogen ( $Sc = 0.22$ ), Oxygen ( $Sc = 0.66$ ), and Carbon dioxide ( $Sc = 0.94$ ).

The effects of governing parameters like magnetic field, chemical reaction, rarefaction parameter, suction parameter, thermal Grashof number as well as mass Grashof number on the transient velocity have been presented in the respective Figures 3 to 8 for both the cases of air and water and in presence of foreign species  $Sc = 0.22$ . In Figure 3, it is observed that an increase in magnetic field has a retarding effect on the velocity of the flow field ( $u$ ) for both the cases of air as well as water in presence of Hydrogen. The presence of transverse magnetic field produces a resistive force on the fluid flow. This force is called the Lorentz force, which leads to slow down the motion of electrically conducting fluid. It is seen from Figure 4 that under the influence of chemical reaction, the flow velocity reduces in air and water both. The hydrodynamics boundary layer becomes thin as the chemical reaction parameter increases. The effect of rarefaction parameter ( $h$ ) on the transient velocity ( $u$ ) is plotted in Figure 5. It is noticed that an increase in rarefaction parameter leads to increase in  $u$ . Also it is

interesting to see that increase in Pr leads to decrease the flow velocity in the interval  $0 \leq y < 2.5$ , while this behaviour is reversed for  $y \geq 2.5$ . It is marked from Figure 6 that the increasing value of the suction parameter reducing the flow velocity for the cases of air and water. However, significantly, it is observed that the flow velocity increases with Pr in the interval  $0 \leq y < 2.5$ , but this behaviour of Pr is opposite for  $y \geq 2.5$ . Moreover, the transient velocity rises with the increasing values Gr is observed in Figure 7, while this velocity decreases for increasing values of Prandtl number. Also it is seen in Figure 8 that, the effect of Gc on the transient velocity is similar to the effect of Gr.

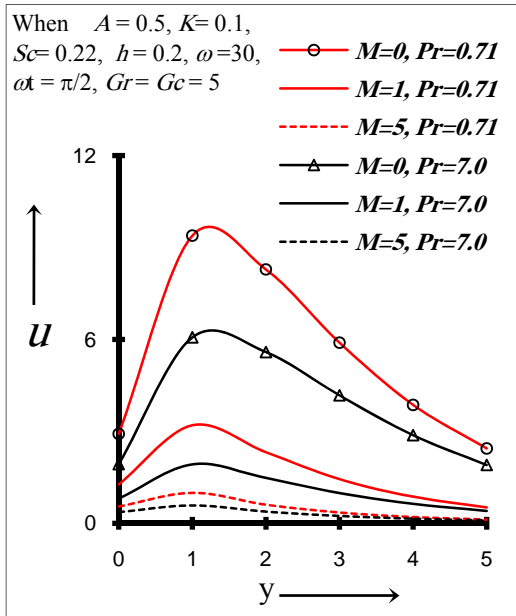


Figure 3 Effect of magnetic parameter on the Transient velocity for Pr = 0.71 and Pr = 7.

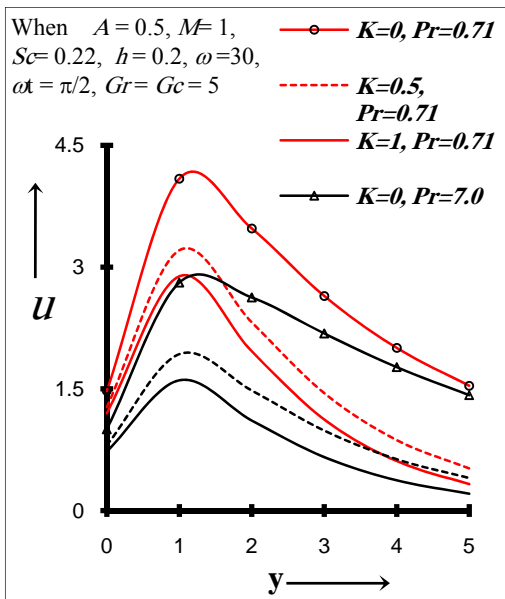


Figure 4 Effect of chemical Reaction parameter on the transient velocity for Pr = 0.71 and Pr = 7.

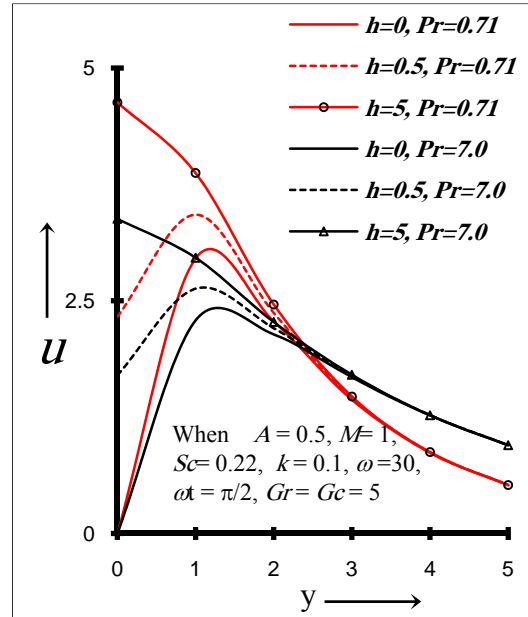


Figure 5 Effect of rarefaction parameter on the transient velocity for Pr = 0.71 and Pr = 7.

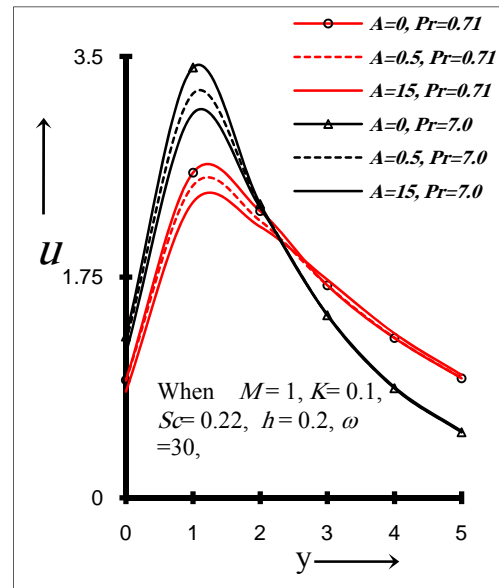


Figure 6 Effect of suction parameter on the transient velocity for Pr = 0.71 and Pr = 7.

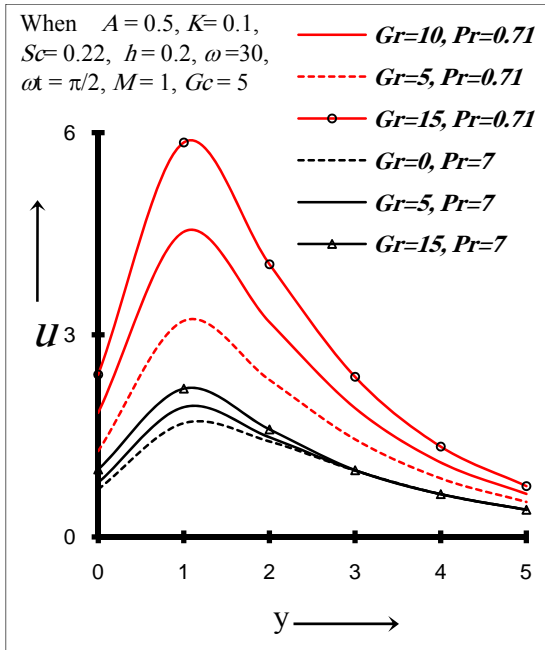


Figure 7 Effect of thermal Grashof number on the transient velocity for  $Pr = 0.71$  and  $Pr = 7$ .

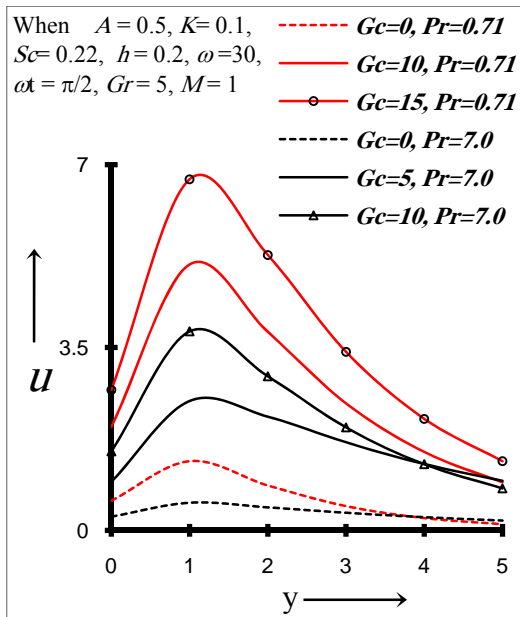


Figure 8 Effect of mass Grashof number on the transient velocity for  $Pr = 0.71$  and  $Pr = 7$ .

Figure 9 illustrate the dimensionless concentration profiles ( $\phi$ ) for chemical reaction ( $K$ ), Schmidt number and suction parameter ( $A$ ) against frequency parameter. A decrease in concentration with increasing  $K$  as well as  $Sc$  is observed from this figure, while the concentration rises with the increase of suction parameter. This is because as the suction rate is increased, more warm fluid is taken away from the boundary layer. Also, it is noted that the concentration boundary layer becomes thin as the Schmidt number as well as chemical reaction parameter increases.

The effects of magnetic field, chemical reaction and rarefaction parameter on the amplitude and phase of the skin-friction are presented in the respective figures 10 and 11. It is observed from Fig. 10 that the influence of  $M$  and  $h$  reduces the amplitude  $|F|$  of skin-friction, whereas increase in chemical reaction leads to increase in amplitude. The phase  $\tan\alpha$  decreases with increasing magnetic field, while this behaviour is opposite to the influence of chemical reaction and rarefaction parameter. It is interesting to observe that the influence of  $M, k$  and  $h$  are insignificant for large values of frequency parameter.

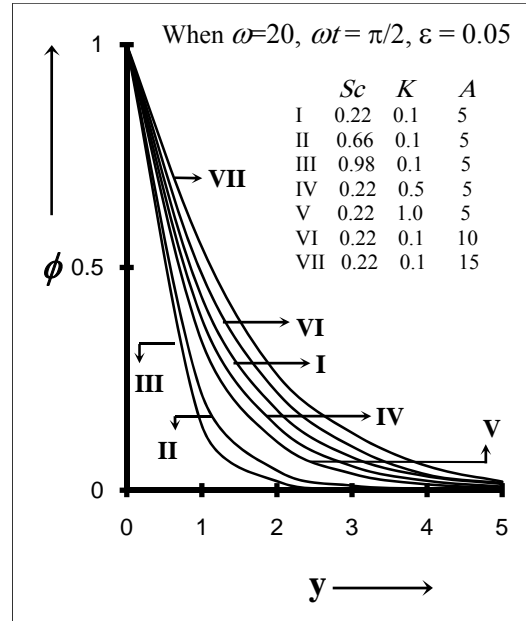


Figure 9 Concentration profile  $\phi$  against  $y$ .

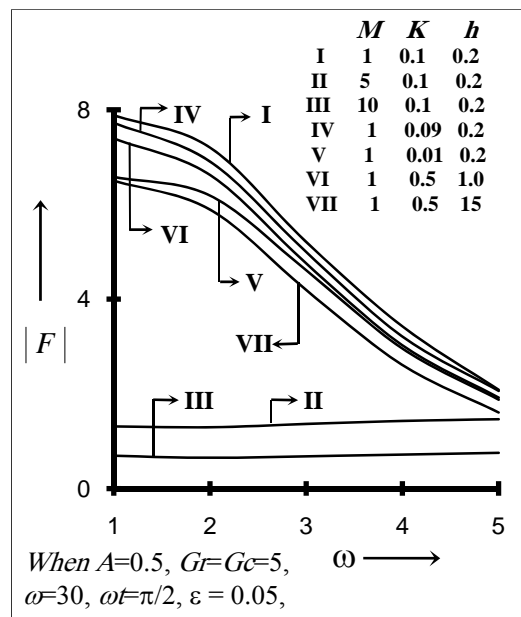


Figure 10 Amplitude of skin-friction  $|F|$  for  $Pr = 0.71$  and  $Sc = 0.94$ .

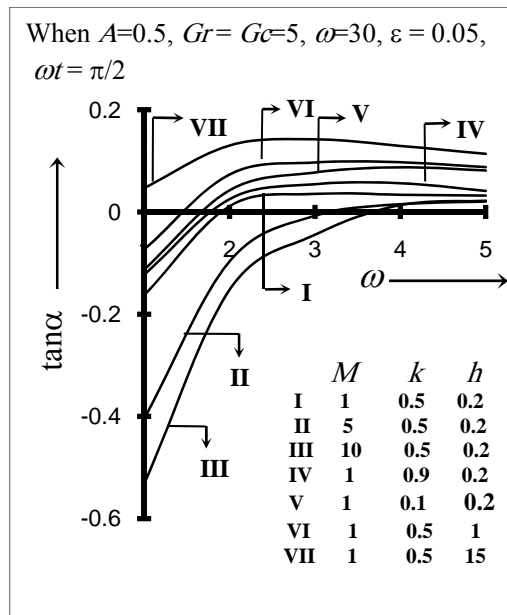


Figure 11: Phase of skin-friction  $|F|$  for  $Pr = 0.71$  and  $Sc = 0.94$ .

### 5. CONCLUSIONS

In this paper, investigations were made on the effects of magnetic field strength, chemical reaction, rarefaction, wall suction as well as combined buoyancies on natural convection periodic heat and mass transfer over a semi-infinite vertical permeable surface in a slip-flow regime. Perturbation technique was employed and graphical results were obtained to illustrate the details of flow characteristics and their dependence on some of the physical parameters. It was found that the magnetic parameter  $M$  has a retarding effect on transient velocity. This is a result of magnetic pull of the Lorentz force acting on the velocity field. However, this same effect was found to decrease both the amplitude as well as phase of skin-friction. The present analysis has shown that the flow is appreciably influenced by chemical reaction. It was observed that, the chemical reaction parameter has a retarding effect on the velocity of flow field as well as concentration distributions. Also it was seen that, the effect of higher Schmidt number results into the thinner concentration boundary layer as higher Schmidt number fluid has a lower concentration diffusivity. The hydrodynamic and concentration boundary layer thickness were observed to decrease as a result of increasing chemical reaction. It was also observed that, with or without thermal and mass buoyancies, there is an accelerating effect on velocity of flow field. It can be seen that for cooling of the plate ( $Gr, Gc > 0$ ), the velocity profiles decrease monotonically with the increase of suction parameter indicating the usual fact that suction stabilizes the boundary layer growth. By sucking the slowed boundary layer material into the inside of the body through narrow slits on the wall boundary layer separation can be prevented.

### Nomenclature

- $(\bar{u}, \bar{v})$  Velocity components along the  $(\bar{x}, \bar{y})$  directions respectively ( $m.s^{-1}$ ),
- $u$  Dimensionless velocity component in x-direction ( $m.s^{-1}$ ),
- $v_0$  Dimensionless suction velocity,
- $\bar{T}$  Temperature (K),
- $\bar{T}_\infty$  Fluid temperature in the free stream (K),
- $\bar{T}_w$  Dimensional temperature at the plate (K),
- $\theta$  Dimensionless fluid temperature (K),
- $\omega$  Frequency of fluctuation ( $S^{-1}$ ),
- $\bar{C}$  Species concentration ( $kg.m^{-3}$ ),
- $\bar{C}_\infty$  Species concentration in the free stream,
- $\bar{C}_w$  Species concentration at the surface,
- $\phi$  Dimensionless species concentration ( $kg.m^{-3}$ ),
- $D$  Chemical molecular diffusivity ( $m^2.s^{-1}$ ),
- $\bar{K}$  Chemical reaction coefficient ( $s^{-1}$ ),
- $K$  Dimensionless Chemical reaction,
- $\tau_x$  Dimensional Shear stress ( $N.m^{-2}$ ),
- $C_p$  Specific heat at constant pressure ( $J.kg^{-1}.K$ ),
- $Pr$  Prandtl number,
- $A$  Suction parameter,
- $Sc$  Schmidt number,
- $t$  time (s),
- $Gr$  Thermal Grashof number,
- $Gc$  Mass Grashof number,
- $h$  rarefaction number,
- $\bar{L}$  Constant,
- $B_0$  Magnetic field flux density,
- $M$  Hartmann number.
- $g$  Acceleration due to gravity ( $m^2.s^{-2}$ ),
- $p$  Pressure ( $N.m^{-2}$ ),

### Greek symbols

- $\alpha$  Thermal diffusivity ( $m^2.s^{-1}$ ),
- $\beta$  coefficient of thermal expansion ( $K^{-1}$ ),
- $\beta$  Coefficient of thermal expansion with concentration ( $K^{-1}$ ),
- $\phi$  Dimensionless species concentration,
- $\kappa$  Thermal conductivity ( $W.m^{-1}.K$ ),
- $\mu$  Coefficient of viscosity (Pa. s),
- $\nu$  Kinematic viscosity ( $m^2.s^{-1}$ ),
- $\theta$  Dimensionless temperature (K),
- $\rho$  density ( $kg.m^{-3}$ ),
- $\sigma$  electrical conductivity ( $\Omega^{-1}.m$ ),
- $\tau$  dimensional shearing stress ( $N.m^{-2}$ ),
- $\omega$  Frequency parameter ( $s^{-1}$ ),



**Subscripts**

w evaluated at wall conditions  
 ∞ evaluated at free stream conditions

**APPENDIX:**

$$n = [Sc + \sqrt{Sc^2 + 4Sc k}] / 2, q = [1 + \sqrt{1 + 4M}] / 2,$$

$$\mu = [Sc + \sqrt{Sc^2 + Sc(i\omega + 4k)}] / 2,$$

$$\lambda = [Pr + \sqrt{Pr^2 + i\omega Pr}] / 2,$$

$$\Psi = [1 + \sqrt{(1 + 4M) + i\omega}] / 2,$$

$$B_1 = 1 - 4AiPr/\omega, B_2 = 1 - B_1,$$

$$B_3 = \frac{Gr}{Pr(Pr-1) - M}, B_4 = \frac{Gc}{n(n-1) - M},$$

$$B_5 = \frac{B_3(hPr+1) + B_4(hn+1)}{q(q-1) - M},$$

$$B_6 = 4iAn/\omega, B_7 = 1 + B_6,$$

$$B_8 = Gr B_2 + APr B_3, B_9 = Gc B_6 - An B_4,$$

$$B_{10} = \frac{Gr B_1}{\lambda^2 - \lambda - i\omega/4 - M},$$

$$B_{11} = \frac{Gc B_7}{\mu^2 - \mu - i\omega/4 - M},$$

$$B_{12} = \frac{Aq B_5}{q^2 - q - i\omega/4 - M},$$

$$B_{13} = \frac{B_8}{Pr^2 - Pr - i\omega/4 - M},$$

$$B_{14} = \frac{B_9}{n^2 - n - i\omega/4 - M},$$

$$B_{15} = [(1 + h\lambda)B_{10} + (1 + h\mu)B_{11} - (1 + hq)B_{12} + (1 + hPr)B_{13} + (1 + hn)B_{14}](1 + h\Psi)^{-1}.$$

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