EFFECTS OF INTERNAL FLAWS IN STATIC AND DYNAMIC BEHAVIOR OF COMPOSITE PLATES

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1. INTRODUCTION AND BACKGROUND

The structural advantages of laminated composite materials are compromised by the presence of defects. The damage can occur during service life, as a result of low-velocity impact or stresses created by excessive loading. The defect alters the dynamic characteristics and also reduces the strength of a structure, particularly under compressive loading[1].

Cawely P. and Adams R. D.[2] developed a method of sensitivity analysis to deduce the location of damage based on the application of the finite element method. The method was applied to a flat isotropic plate. The sensitivity of the change in eigenvalue was evaluated for each of the element in the model. The results of the analysis gave good agreement with the experimental findings and useful indications of the magnitude of the defect were obtained.

The crack problem for orthotropic plates under bending and membrane loading was considered by Bing-Hua Wu and Erdogan F.[3]. The surface crack problem is formulated by using the line spring model with a transverse shear theory of plate bending. Examples are given for composite laminates containing one or two semielliptic surface cracks and subjected to membrane loading or bending.

The aim of the work presented by Ostachowicz W. M. and Krawczuk M.[4] was to analyze the influence of a fatigue crack and delamination on the changes in the dynamics of structures made of composite materials. This problem was solved by using the finite element method. The damaged part of the structures was modeled by special finite elements with failures, while the undamaged part was represented by other, well known finite elements. The influence of material parameters (fiber angle and volume of the fiber) on the intensity of the change was also investigated.

Roderick H. Martin[5] investigated delamination failure in a unidirectional curved composite laminate. The curved laminate failed by delamination developing around the curved region of the laminate at
different depths through the thickness until virtually all bending stiffness was lost. Delamination was assumed to initiate at the location of the highest radial stress in the curved region. A closed form curved beam elasticity solution and a 2-D finite element analysis (FEA) were conducted to determine this location.

The line spring model and its implementation in the finite element program ABAQUS was reviewed by Goncalves J. P. M. and de Castro P. M. S. T. [6]. The line spring capabilities in that program are used to analyze some part-through crack configurations, namely a semi-elliptic surface crack a quarter-circular corner crack at a hole and an example of a complex damage scenario.

Many researches investigated the effect of defects by considering delamination, matrix cracking and fiber-matrix interface without involving the effect of voids. The objectives of this study are to investigate the static and dynamic (free and transient response) behaviour of general laminated plates with and without defect (internal and surface flaws voids). Also, the influence of the size and location of the defects is considered in this work. Theoretical and experimental approaches are used in this work. In the experimental work a number of specimens are produced with different number of layers and different boundary conditions to simulate the defect in composite material and measuring natural frequency with and without defect. The numerical approach involved developing a finite element program to simulate composite materials with and without defect using two model of defect.

2. EXPERIMENTAL WORK

2.1 Test specimens design and Preparation:

The plate strips specimen used in the investigation were fabricated from the unidirectional E-glass and epoxy plies. The symmetrical lay – up were prepared by using the wood open mould of thick wood plates. The molder applies a pigments “release” material to the mould, as the First step in making any open mould product. Without such material, the part will permanently bond to the mould surface. The second step is to mix thoroughly pre – measured unsaturated epoxy and catalyst together, and for ensuring complete air removal and wet out, the mixture should cover the base surface completely especially at the end edges. The Third step is the application of E-glass fiber in the resin layer with the desired orientation θ. These fibers are connected manually to needles to build the required fiber mat as shown in figure (1). Then, a new fiber mat at another orientation angle is built using the same procedure. The process is repeated until the total design number of layers is achieved. A quantity of epoxy resin (depending on volume fraction) is then added over the fibers and compacted using a suitable hand roller. The result composite plate is shown in figures (2).

2.2 Tensile test

The mechanical properties of composite plate obtained from the tensile test, the specimens have the standard dimensions [7]. The result tensile test are listed in table (1)

<table>
<thead>
<tr>
<th>E1 (Gpa)</th>
<th>E2 (Gpa)</th>
<th>G12 (Gpa)</th>
<th>v12</th>
<th>v21</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.618</td>
<td>5.25</td>
<td>2.52</td>
<td>0.21</td>
<td>0.04</td>
</tr>
</tbody>
</table>

2.3 Experimental representation of damage:

The first step is to fix the specimen to the frame structure as shown in figure (3), the construction depends on the boundary conditions which are required. This is followed by connecting the different instruments device and the specimen as shown in figure (3).

The second step is the measuring of the natural frequency without defect for first and second mode, then, a drill was used to make holes in the plate with diameter equal to (10mm) and the first and second natural frequency is measured again. After that, holes are increased and at each increases of holes number the natural frequency is measured. The arrangement of increasing holes defect as well as dimensions of the plates and holes are shown in figure (4).
3. NUMERICAL SOLUTION

3.1. Special Third-Order Theory (HOST 7): -

It is a special case of the general third-order theory, which presents displacement components given in the form[9]:

\[ u(x,y,z,t) = u_0(x,y,t) + z \phi_x(x,y,t) + z^3 \theta_x(x,y,t) \]
\[ v(x,y,z,t) = v_0(x,y,t) + z \phi_y(x,y,t) + z^3 \theta_y(x,y,t) \]
\[ w(x,y,z,t) = w_0(x,y,t) \]  (1)

The strain-displacement and stress-strain relations are explained in[9]:

\[ \epsilon = \sigma D \]  (4a)

3.2. Formulation of Elasticity Matrix of Composite Laminated Plates

In the following procedure, the elasticity matrix \([D]\) is evaluated based on the special third order shear deformation theory. According to the third order shear deformation theory, the stress resultants are defined as follows:

\[ \begin{bmatrix} N_x & M_x & M_{xy}^* \\ N_y & M_y & M_{xy}^* \\ N_{xy} & M_{xy} & M_{xy}^* \end{bmatrix} = \sum_{L=1}^{N} \int_{h_1}^{h_L} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} (t_1, z, z^3) dz \]  (2)

\[ \begin{bmatrix} Q_x & Q_x^* \\ Q_y & Q_y^* \end{bmatrix} = \sum_{L=1}^{N} \int_{h_1}^{h_L} \begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} (1, z^2) dz \]  (3)

By substituting stresses vectors in equs. (2 and 3) and integrating with respect to \(z\), the stress-resultants are obtained in terms of the seven generalized displacements as[9]:

\[ \{\tau\} = [D] \{\epsilon\} \]  (4a)
Where:

\[
\begin{bmatrix}
N_1 \\
N_2 \\
N_3 \\
N_4 \\
N_5 \\
N_6 \\
N_7 \\
M_1 \\
M_2 \\
M_3 \\
M_4 \\
M_5 \\
M_6 \\
M_7 \\
M_{xy} \\
M_{yx}
\end{bmatrix}
= \frac{8}{L^4} \mathbf{I}
\]

\[
\begin{bmatrix}
Q_{x1} \\
Q_{x2} \\
Q_{x3} \\
Q_{x4} \\
Q_{x5} \\
Q_{x6} \\
Q_{x7} \\
Q_{y1} \\
Q_{y2} \\
Q_{y3} \\
Q_{y4} \\
Q_{y5} \\
Q_{y6} \\
Q_{y7} \\
Q_{xy} \\
Q_{yx}
\end{bmatrix}
= \sum_{i=1}^{9} \mathbf{Q}_i
\]

\[
\mathbf{M}_{xy} = \text{SYMMETRIC}
\]

\[
\begin{bmatrix}
Q_{x1} \\
Q_{x2} \\
Q_{x3} \\
Q_{x4} \\
Q_{x5} \\
Q_{x6} \\
Q_{x7} \\
Q_{y1} \\
Q_{y2} \\
Q_{y3} \\
Q_{y4} \\
Q_{y5} \\
Q_{y6} \\
Q_{y7} \\
Q_{xy} \\
Q_{yx}
\end{bmatrix}
= \sum_{i=1}^{9} \mathbf{Q}_i
\]

\[
\begin{bmatrix}
\frac{\partial^2}{\partial x^2} \delta_1 \\
\frac{\partial^2}{\partial y^2} \delta_1 \\
\frac{\partial^2}{\partial x \partial y} \delta_1 \\
\frac{\partial^2}{\partial y \partial x} \delta_1
\end{bmatrix}
\]

where:

\[
H_i = \frac{1}{i} (h_i^{1} - h_i^{2}) \quad i = 1, 2, 3, 4, 5, 7
\]

### 3.2.1 Element Stiffness Matrix

A nine-node Lagrangian quadrilateral element is used in this work for the discretization of fiber-reinforced laminated plates. This element contains four nodes at the corners, four nodes at the mid-side of the element boundaries and one node at the center of the element. The shape functions of this element are given by [9], as:

\[
N_1 = 1 - 3(\zeta + \eta) + 9(\zeta^2 + \eta^2) - 6(\zeta^2 + \zeta \eta) + 4(\zeta^2 + \eta^2)
\]

\[
N_2 = 4(\zeta - \zeta^2) + 12(\zeta^2 - \zeta \eta) + 8(\zeta^2 - \zeta \eta^2)
\]

\[
N_3 = 3\zeta - \zeta + 2(\zeta^2 - \zeta \eta) - 6\zeta^2 + 4\zeta \eta
\]

\[
N_4 = 4(\zeta^2 - \zeta \eta) + 8(\zeta^2 - \zeta \eta^2)
\]

\[
N_5 = \zeta - 2(\zeta^2 + \zeta \eta) + 4\zeta \eta^2
\]

\[
N_6 = 4(\zeta^2 - \zeta \eta) + 8(\zeta^2 - \zeta \eta^2)
\]

\[
N_7 = 3\zeta - \zeta + 2(\zeta^2 - \zeta \eta) - 6\zeta^2 + 4\zeta \eta
\]

\[
N_y = 4(\zeta^2 - \zeta \eta) + 12(\zeta^2 - \zeta \eta^2) + 8(\zeta^2 - \zeta \eta^2)
\]

\[
N_9 = 16(\zeta^2 - \zeta \eta + \zeta^2 \eta^2)
\]

where \(\zeta\) and \(\eta\) have the values between \((0 \leq \pm 1)\).

At any point, the continuum displacement vector within the element is discretized such that:

\[
\{\delta\} = \sum_{i=1}^{9} N_i \{\delta_i\}
\]

where:

\[
\{\delta_i\} = \{u_{i1}, v_{i1}, w_{i1}, \phi_{xi}, \phi_{yi}, \theta_{xi}, \theta_{yi}, \phi_{xi}, \phi_{yi}\}^T
\]

The element stiffness matrix is given by:

\[
[K] = \int_{A} \{B\}^T [D] \{B\} \text{d}A
\]

The strain-displacement matrix \([B]\) are given as [9]:

\[
B_{11} = B_{32} = B_{44} = B_{6,5} = B_{7,6} = B_{9,7} = B_{10,3} = \frac{\partial N_1}{\partial x}
\]

\[
B_{10,4} = B_{11,5} = N_1
\]

\[
B_{2,2} = B_{31} = B_{5,5} = B_{6,4} = B_{8,7} = B_{9,6} = B_{11,3} = \frac{\partial N_1}{\partial y}
\]

\[
B_{12,6} = B_{13,7} = 3N_1
\]

### 3.2.2 Element Mass Matrix

A mass matrix is derived from a consistent matrix. It is called “Consistent” because the same displacement mode, used for deriving the element stiffness matrix, is used for the derivation of mass matrix. To derive the consistent mass matrix, consider the kinetic energy of the total solution domain discretized into \((N_e)\) elements.

\[
TI(\dot{\delta}) = \sum_{i=1}^{N_e} TI' (\dot{\delta})
\]

In this work, the mass matrix of fiber-reinforced laminated plates is derived from a consistent mass matrix. The mass \([M]^c\) is:

\[
[M]^c = \int_{A} \{N\}^T [m] \{N\} \text{d} (\text{Area})
\]

### 4. DEFECT MODELING

The defect in multi layer composite plate may occur in any layers as a flaw due to manufacturing and this defect will decrease the stiffness of the plate, which depends on the size and location of the defect.
Therefore, two models of defect in composite plate are suggested, first model is removing layers model (In this model, the $Q$ matrix for the layer in any element with defect will be diminished. This model can be used for modeling the voids in composite laminated plate), second model is weakness Layers Model (in this model the $Q$ matrix for the element which has the defect in any layer is multiplied by a factor such as (0.1, 0.2, etc) depending on the weakness degree of the layer).

5. NUMERICAL RESULT

In the following work, it is assumed that the material is fiber-reinforced and remains in the elastic range. The boundary conditions are simply supported (unless other boundary conditions are mentioned), and the numerical procedure (HOST 7) is used depending on a finite element package developed in this work. The material properties are\(^{[10]}\):

$E_1=(10 \text{ to } 40) \ E_2$, $G_{12}=G_{13}= 0.5 \ E_2$, $G_{23}=0.6E_2$, $v_{12}=0.25$

The dimensions of plates are: $-a=0.25\text{m}$, $b=0.25\text{m}$, $h=10\text{mm}$, Load:-distributed load=1 kps

Figure (5) shows the relationship between the percentage of defect area and maximum deflection for angle ply (layer 1 and layer 2). From this figure, it can be seen that, increasing area of defect increases maximum deflection by (55%) when the percentage of defect area is (52%) for the top layer (layer number 1). At the same defect area percentage; the maximum deflection is increased by 18% for the middle layer (layer number 2).

For cross ply figure (6) shows the maximum deflection increases by (56.3%) when the defect area is (52%) for the top layer and (17%) for the middle layer. Figure (7) is for angle ply with $E_1/E_2=40$. The maximum deflection is increased by (110%) for layer number (1) and (41%) for layer number (2) when the defect area is (52%). Figure (8) is for cross ply with $E_1/E_2=40$. The maximum deflection is increased by (61%) for layer number (1) and (64%) for layer number (2).

The effect of weakness percentage of element and defect area percentage on the maximum deflection of a square antisymmetric angle laminated plate is shown in figure (9). From this figure, the maximum deflection is increased with the increases in weakness factor from 25% to 100 % defects also increasing the defect area increases maximum deflection.

The same behavior is indicated in figure (10) for cross laminated plate. It can be concluded that, the increasing weakness factor increases maximum deflection due to the decrease in the stiffness of the
plate, and the maximum effect occurs when the defect is full (weakness factor=100%) because the stiffness of plate will be at its smallest value.

Figures (11,12) show the relationship between the defect area percentage and the natural frequency for angle ply and cross ply respectively (layer 1 and layer 2). From these figures, increasing the area of defect decreases the natural frequency. Also, it is seen that the top layer gives more effect in natural frequency compared to the middle layer.

From above figures, increasing the area of defect decreases the stiffness matrix and the mass matrix of plate, but the decrease in mass matrix is greater than the stiffness matrix therefore the natural frequency is decreased.

Figure (13) shows the effect of weakness percentage of element and defect area percentage on a non-dimensional frequency of a square antisymmetric angle laminated plate, the natural frequency is decreased with the increase in weakness factor from 25% to 100% defect also increasing area of defect decreases natural frequency.

Similarity in Figure (14) for the effect of weakness percentage of element and defect area percentage on the natural frequency of a square antisymmetric cross laminated plate. From above, increasing weakness factor decreases natural frequency due to decreasing the stiffness of plate, and the maximum effect occurs when the defect is full (weakness factor=100%) because the stiffness of plate will be at its smallest value.
6. DYNAMIC RESULT

Figure (15) shows the effect of percentage of defect area on the central transverse deflection of square plates (45/-45/45/-45) under uniform pulse load (E1/E2=10). From this fig, it is seen that increasing the area of defect causes an increase in the maximum amplitude due to the decrease stiffness of plate. The same behaviour is repeated in fig (16) but for a square cross-ply laminated plate (0/90/0/90). Here, also the maximum deflection is increased with the increasing in defect area.

From above, cross-ply arrangement of layers gives maximum deflection greater than the maximum deflection in the angle-ply arrangement of layers this is due to the stiffness in the angle-ply arrangement being greater than that in the cross-ply arrangement.

Figure (17) shows the effect of percentage of defect area percentage on the central transverse deflection of square plates (45/-45/45/-45) under uniform triangle load (E1/E2=10). It is seen that, increasing the area of defect causes an increase in the maximum deflection of plate. Figure (18) shows the same behaviour as above but for square cross-ply plates (0/90/0/90).

From (19) gives the effect of defect area percentage on the central transverse deflection of square plates (45/-45/45/-45) under sinusoidally distributed load (E1/E2=10). It is seen that, increasing the area of defect causes an increase in the maximum deflection of plate. Figure (20) shows the same behaviour as above but for square cross-ply plates (0/90/0/90).
Figure 20. Effect of defect area percentage on the central transverse deflection of square plates (0/90/0/90) under sinusoidally distributed sine load. (E1/E2=10)

Figure (21) shows the effect of defect area percentage on the central transverse deflection of square plates (45/-45/45/-45) under sinusoidally distributed ramp load (E1/E2=10). It is seen that, increasing the area of defect causes an increase in the maximum deflection of plate. Figure (22) shows the same behaviour as above but for square cross-ply plates (0/90/0/90).

7. EXPERIMENTAL RESULT

The experimental work in this study depends on measuring natural frequency with different boundary conditions, different lamination angles, and different number of layers. The material properties are listed in table (1). The dimensions of the specimens are: a=b=25 cm, thickness (layer) =0.15 cm, hole diameter=1 cm.

Tables (2, 3 and 4) give the relation as above but now for three layers cross ply (0/90/0) and angle ply (45/-45/45). From tables, good agreements have been obtaining between experimental work and numerical work.

<table>
<thead>
<tr>
<th>Number of Holes</th>
<th>0/90/0</th>
<th>45/-45/45</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Num. result</td>
<td>Exper. result</td>
</tr>
<tr>
<td>0</td>
<td>403.42</td>
<td>330.5</td>
</tr>
<tr>
<td>1</td>
<td>363.49</td>
<td>295.3</td>
</tr>
<tr>
<td>2</td>
<td>372.77</td>
<td>298.3</td>
</tr>
<tr>
<td>3</td>
<td>359.16</td>
<td>280.3</td>
</tr>
<tr>
<td>4</td>
<td>362.0</td>
<td>296.6</td>
</tr>
<tr>
<td>5</td>
<td>358.09</td>
<td>285.3</td>
</tr>
</tbody>
</table>
Table 3 Comparison between experimental work and Numerical work for natural frequency (Hz), of three layer laminated plates having different defect area with CCCF boundary condition

<table>
<thead>
<tr>
<th>Number of Holes</th>
<th>0/90/0 First mode</th>
<th>0/90/0 Second mode</th>
<th>45/-45/-45 First mode</th>
<th>45/-45/-45 Second mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Num. result</td>
<td>Exper. result</td>
<td>Discrepancy %</td>
<td>Num. result</td>
</tr>
<tr>
<td>0</td>
<td>242.47</td>
<td>201.3</td>
<td>16.97</td>
<td>388.48</td>
</tr>
<tr>
<td>1</td>
<td>242.74</td>
<td>195.65</td>
<td>19.39</td>
<td>390.67</td>
</tr>
<tr>
<td>2</td>
<td>244.75</td>
<td>206.5</td>
<td>15.62</td>
<td>401.55</td>
</tr>
<tr>
<td>3</td>
<td>233.17</td>
<td>195.7</td>
<td>16.75</td>
<td>403.1</td>
</tr>
<tr>
<td>4</td>
<td>243.87</td>
<td>192.91</td>
<td>20.89</td>
<td>390.4</td>
</tr>
</tbody>
</table>

Table 4 Comparison between experimental work and Numerical work for natural frequency (Hz) of three layer laminated plates having different defect area with CCFF boundary condition

<table>
<thead>
<tr>
<th>Number of Holes</th>
<th>0/90/0 First mode</th>
<th>0/90/0 Second mode</th>
<th>45/-45/-45 First mode</th>
<th>45/-45/-45 Second mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Num. result</td>
<td>Exper. result</td>
<td>Discrepancy %</td>
<td>Num. result</td>
</tr>
<tr>
<td>0</td>
<td>69.71</td>
<td>58.65</td>
<td>15.86</td>
<td>246.81</td>
</tr>
<tr>
<td>1</td>
<td>72.22</td>
<td>55.5</td>
<td>23.15</td>
<td>230.73</td>
</tr>
<tr>
<td>2</td>
<td>71.49</td>
<td>53.08</td>
<td>25.75</td>
<td>227.01</td>
</tr>
<tr>
<td>3</td>
<td>71.24</td>
<td>52.12</td>
<td>26.83</td>
<td>232.35</td>
</tr>
<tr>
<td>4</td>
<td>68.84</td>
<td>56.85</td>
<td>17.41</td>
<td>238.65</td>
</tr>
<tr>
<td>5</td>
<td>65.304</td>
<td>50.22</td>
<td>23.09</td>
<td>227.26</td>
</tr>
</tbody>
</table>

7. CONCLUSIONS

The main conclusions of this work for both static and dynamic analyses of multilayer composite plates may be summarized as:

1. The increasing surface area of defect causes an increase in the maximum deflection and the maximum value of transient deflection, also, decreases the natural frequency.

2. The weakness model of defect showed that; the increase in the percentage of weakness of element causes an increase in the maximum deflection and a decrease in the natural frequency.

3. The defect in top and bottom layers decreases the stiffness of plate more than the defect inside the plate. Therefore, the displacement is increased and the natural frequency is decreased when the defect occurs in top and bottom layers.

4. For an angle-ply laminated plate, it is found that; \((\theta=45^\circ)\) represents the best lamination angle at which the deflection is a minimum and the natural frequency is a maximum

REFERENCES


List of Symbols

A Area integral indicator.
[Bij] Strain-displacement matrix.
CCCC Clumped from four edges.
CCCF Clumped from three edges and other edge is free.
CCFF Clumped from two edges and other two edges are free.
[D] Elasticity matrix of laminated plates.
E1 Young modulus of lamina in 1-direction (N/ m²).
E2 Young modulus of lamina in 2-direction (N/ m²).
G12 Shear modulus in the 1-2 plane (N/ m²).
h Laminate thickness (m).
hi Thickness of ith layer (m).
hL,hL+1 Distance from plate middle surface to the lower and upper surface of ith layer respectively.
I1, I2, and I3 Inertia terms of laminates plate.
k kg/m², kg/m, and kg respectively. Subscript refers to layer number.
[K] Stiffness matrix.
[M] Laminate mass matrix.
Mx, My, Mxy Resultant Moments per unit length (N. m / m).
Mx*, My*, Mxy* High-order stress-resultants (N. m³).
[N] Shape function matrix.
N Number of laminate’s layers.
N₁, N₂, N₃ Number of nodes per element.
Nₓ, Nᵧ, Nxᵧ Resultant forces per unit length (N/m).
[Q] Stress-strain relation in principal material directions.
Qₓ, Qᵧ Transformed stress-strain relation (N/m²).
Qₓ₁, Qᵧ₁ High-order shear force in the normal faces to the x and y axis (N/m).
Qₓ₂, Qᵧ₂ High-order shear force in the normal faces to x and y-axis (N. m).
T Energy (joule).
Uₑ Element virtual strain energy of internal stresses (joule).
u, v, w Displacement in the x, y and z directions (m).
uₒ, vₒ, wₒ Middle surface displacement components in x, y and z directions (m).
S, F, C Simply supported, free, and clamped edge conditions.
x, y, z Rectangular coordinate.
{δ} Element nodal displacement vector.
{δᵢ} Vector of degree of freedom in node i.
ρ(k) Material mass density of layer k (kg/m³).
σ Normal stress (N/m²).
τ Shearing stress (N/m²).
Π Potential energy.
φᵢ Cross sectional rotations of the transverse normal, I may be 1, 2, 3 or x, y, z.
θᵢ Higher order transverse cross section deformation modes, I may be x, y.
ω Non-dimensional natural frequency.
v Poisson’s ratio.