MHD AND RADIATIVE FLOW PAST AN VERTICAL OSCILLATING PLATE WITH CHEMICAL REACTION OF FIRST ORDER

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Hydromagnetic and thermal radiation effects on unsteady free convective flow of a viscous incompressible flow past an infinite isothermal vertical oscillating plate with variable mass diffusion are presented here, taking into account the homogeneous chemical reaction of first order. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised to a fixed value, when the plate is oscillating harmonically in its own plane. The effects of velocity, temperature and concentration are studied for different parameters like magnetic field parameter, phase angle, Schmidt number, thermal radiation parameter, chemical reaction parameter, thermal Grashof number, mass Grashof number and time. It is observed that the velocity increases with decreasing phase angle or radiation parameter. It is also observed that the velocity increases with decreasing phase angle \( \omega \) or chemical reaction parameter.

Key words : chemical reaction, radiation, oscillating, vertical plate, magnetic field.

1. INTRODUCTION

Magnetohydrodynamics plays an important role in agriculture, petroleum industries, geophysics and astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earths core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al [1]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al [2]. The dimensionless governing equations were solved using Laplace transform technique.

The Effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. Chambre and Young [3] have analyzed a first
order chemical reaction in the neighbourhood of a horizontal plate. Das et al.[4] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al.[5]. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. England and Emery[6] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar[7]. The governing equations were solved analytically. Das et al[8] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar[9]. The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar[10]. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar et al.[11].

However the combined study of MHD and thermal radiation effects on infinite oscillating isothermal vertical plate with variable mass diffusion, in the presence of chemical reaction of first order is not studied in the literature. It is proposed to study the chemical reaction effects on unsteady flow past infinite isothermal vertical oscillating plate, in the presence of magnetic field and thermal radiation. The dimensionless governing equations are tackled using the Laplace transform technique and the resultant solutions are in terms of exponential and complementary error function.

2. BASIC EQUATIONS AND ANALYSIS

Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature $T_\infty$ and concentration $C'_\infty$. Here, the $x$-axis is taken along the plate in the vertically upward direction and the $y$-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency $\omega'$ and the temperature of the plate is raised to $wT_\infty$ and the concentration level near the plate are also raised linearaly with time $'t$. The plate is also subjected to a uniform magnetic field of strength $B_0$. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. The physical model of the problem shown in figure 1.
Then by usual Boussinesq's approximation, the unsteady flow is governed by the following:

\[ \frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta'(C^* - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \]

(1)

\[ \rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \]

(2)

\[ \frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_C C' \]

(3)

In most cases of chemical reactions, the rate of reaction depends on the concentration of the species itself. A reaction is said to be of the order \( n \), if the reaction rate if proportional to the \( n \)th power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

With the following initial and boundary conditions:

\[ t' \leq 0: \ u = 0, \quad T = T_\infty, \quad C = C'_\infty \text{ for all } y \]

\[ t' > 0: \ u = u_0 \cos \omega t', \quad T = T_\infty, \quad C = C'_\infty + (C'_\infty - C'_\infty) \text{ at } y = 0 \]

\[ u = 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C'_\infty \text{ as } y \rightarrow \infty \]

(4)

Where, \( A = \frac{u_0^2}{v} \)

The local radiant for the case of an optically thin gray gas is defined as

\[ \frac{\partial q_r}{\partial y} = -4a^* \sigma(T_\infty^4 - T^4) \]

It is assume that the temperature differences within the flow are sufficiently small such that \( T^4 \) may be expressed as a linear function of the temperature. This is accomplished by expanding \( T^4 \) in a Taylor series about \( T_\infty \) and neglecting higher-order terms, thus

\[ T^4 \cong 4T_\infty^3 (T - 3T_\infty^4) \]

(6)

By using equations (5) and (6), equation (2) reduces to

\[ \rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3 (T - T_\infty) \]

(7)

On introducing the following non-dimensional quantities:

\[ U = \frac{u}{u_0}, \quad I = \frac{t' u_0^2}{v}, \quad Y = \frac{u y_0}{v}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \]
in equations (1), (3) and (7), reduces to
\[
\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} - M U \tag{9}
\]
\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr} \theta \tag{10}
\]
\[
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - K C \tag{11}
\]

The initial and boundary conditions in non-dimensional form are
\[
U = 0, \quad \theta = 0, \quad C = 0, \quad \text{for all} \quad Y, t \leq 0
\]
\[
t > 0: \quad U = \text{cos}\alpha, \quad \theta = 1, \quad C = t, \quad \text{at} \quad Y = 0
\]
\[
U = 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty
\]  

The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction. The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

\[
\theta = \frac{1}{2} \left[ \text{exp}(2\eta \sqrt{Rt}) \text{erfc}(\eta \sqrt{Pr} + \sqrt{at}) + \text{exp}(-2\eta \sqrt{Rt}) \text{erfc}(\eta \sqrt{Pr} - \sqrt{at}) \right] \tag{13}
\]

\[
C = \frac{t}{2} \left[ \text{exp}(2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{Kt}) + \text{exp}(-2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) \right] - \frac{\eta \sqrt{Sc}}{2\sqrt{K}} \left[ \text{exp}(-2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) - \text{exp}(2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{Kt}) \right] \tag{14}
\]

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction. The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:
\[ + e \exp(ct) \left[ \exp\left(-2\eta\sqrt{Sc(K+c)t}\right) \text{erfc}\left(\eta\sqrt{Sc} - \sqrt{(K+c)t}\right) \right] \\
+ \exp\left(2\eta\sqrt{Sc(K+c)t}\right) \text{erfc}\left(\eta\sqrt{Sc} + \sqrt{(K+c)t}\right) \]  
\hspace{1cm} (15)

Where,
\[ a = \frac{R}{Pr}, b = \frac{R}{1 - Pr}, c = \frac{M - KSc}{1 - Sc}, d = \frac{Gr}{2b(1 - Pr)}, e = \frac{Gc}{2c^2(1 - Sc)}. \]
\[ \eta = Y/2\sqrt{t} \] and erfc is called complementary error function.

3. DISCUSSION OF RESULTS

The numerical values of the velocity and skin-friction are computed for different parameters like Magnetic field parameter, chemical reaction parameter, Schmidt number, thermal Grashof number and mass Grashof number. The purpose of the calculations given here is to assess the effects of the parameters \( M, K, Gr, Gc \) and \( Sc \) upon the nature of the flow and transport. The value of Prandtl number \( Pr \) is chosen such that they represent water \( (Pr = 7.0) \). The solutions are in terms of exponential and complementary error function.

Figure 2 illustrates the effect of the concentration profiles for different values of the chemical reaction parameter \( K = 0.2, 2, 5, 10 \) at \( t = 0.4 \). The effect of chemical reaction parameter is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the velocity increases with decreasing chemical reaction parameter.
The temperature profiles are calculated for different values of thermal radiation parameter \( (R = 2, 5, 7, 10) \) at time \( t = 0.4 \) and these are shown in figure 3. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

![Figure 3. Temperature profiles for different R](image)

The velocity profiles for different phase angles \( (\omega t = 0, \pi/6, \pi/3, \pi/2) \), \( R = 5, M = 2, K = 2, Gr = Gc = 5 \) are shown in figure 4.

![Figure 4. Velocity profiles for different values of \( \omega t \)](image)
It is observed that the velocity increases with decreasing phase angle $\omega t$. Figure 5 demonstrates the effects of the magnetic field parameter on the velocity when $(R = 0.2, 5, 20)$, $\omega t = \pi/6, Gr = Gc = 5, M = 5, K = 2$ and $t = 0.4$. It is observed that the velocity increases with decreasing magnetic field parameter.

![Figure 5. Velocity profiles for different values of R](image)

Figure 5. Velocity profiles for different values of R

Figure 6 illustrates the effect of the velocity for different values of the reaction parameter $(K = 0.2, 7, 15), \omega t = \pi/6, R = 5, M = 2, Gr = Gc = 5$ and $t = 0.6$. The trend shows that the velocity increases with decreasing chemical reaction parameter. The effect of velocity for different values of the radiation parameter $(M = 0.2, 2, 5), \omega t = \pi/6, K = 2, M = 5, Gr = Gc = 5$ and $t = 0.4$ are shown in figure 7. The trend shows that the velocity increases with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation.
Figure 6. Velocity profiles for different values of $K$

Figure 7. Velocity profiles for different values of $M$
4. CONCLUSION

Theoretical solution of hydromagnetic and thermal radiation effects on flow past an oscillating infinite isothermal vertical plate with variable mass diffusion, in the presence of chemical reaction of first order. The dimensionless equations are solved using Laplace transform technique. The effect of velocity, temperature and concentration for different parameters like $\omega t, M, R, K, Gr, Gc, Sc$ and $t$ are studied. The study concludes that the velocity increases with decreasing magnetic field parameter $M$ or radiation parameter $R$. It is also observed that the velocity increases with decreasing phase angle $\omega t$ or chemical reaction parameter $K$.

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Nomenclature

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>constant</td>
</tr>
<tr>
<td>$a^*$</td>
<td>absorption coefficient</td>
</tr>
<tr>
<td>$B_0$</td>
<td>external magnetic field</td>
</tr>
<tr>
<td>$C'$</td>
<td>species concentration in the fluid</td>
</tr>
<tr>
<td>$C$</td>
<td>dimensionless concentration</td>
</tr>
<tr>
<td>$C_p$</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>$D$</td>
<td>mass diffusion coefficient</td>
</tr>
<tr>
<td>$Gr$</td>
<td>mass Grashof number</td>
</tr>
<tr>
<td>$Gc$</td>
<td>thermal Grashof number</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity</td>
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<tr>
<td>$K_t$</td>
<td>chemical reaction parameter</td>
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<tr>
<td>$K$</td>
<td>dimensionless chemical reaction parameter</td>
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<tr>
<td>$M$</td>
<td>magnetic field parameter</td>
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<td>$Pr$</td>
<td>Prandtl number</td>
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<td>$R$</td>
<td>Radiation parameter</td>
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<tr>
<td>$Sc$</td>
<td>Schmidt number</td>
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<tr>
<td>$T$</td>
<td>temperature of the fluid near the plate</td>
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<td>$t'$</td>
<td>dimensionless time</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity of the fluid in the $x'$-direction</td>
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<td>$u_0$</td>
<td>velocity of the plate</td>
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<td>$u$</td>
<td>dimensionless velocity</td>
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<tr>
<td>$y$</td>
<td>coordinate axis normal to the plate</td>
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<tr>
<td>$Y$</td>
<td>dimensionless coordinate axis normal to the plate</td>
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Greek symbols

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<tr>
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<td>thermal diffusivity</td>
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<td>volumetric coefficient of expansion with concentration</td>
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<tr>
<td>$\mu$</td>
<td>coefficient of viscosity</td>
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<tr>
<td>$\nu$</td>
<td>kinematic viscosity</td>
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<td>$\omega t$</td>
<td>phase angle</td>
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<td>density of the fluid</td>
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<td>dimensionless skin-friction</td>
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<td>$\eta$</td>
<td>dimensionless temperature</td>
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<tr>
<td>$\eta$</td>
<td>similarity parameter</td>
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<tr>
<td>$\text{erfc}$</td>
<td>complementary error function</td>
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Subscripts

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<td>$\omega$</td>
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</tr>
<tr>
<td>$\infty$</td>
<td>conditions in the free stream</td>
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REFERENCES

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