PERFORMANCE EVALUATION OF SPECTRAL EFFICIENCY IN LINEAR REGIME FOR HIGH RATE WDM SYSTEM

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This paper addresses the effect of various system parameters on the spectral efficiency of wavelength division multiplexing (WDM) systems. Three types of WDM systems were investigated: unconstrained modulation with coherent detection (UMCD) system, constant-intensity modulation with coherent detection (CIMCD) system, and unconstrained modulation with the direct detection (UMDD) system. Expressions are derived and presented to assess the spectral efficiency of these systems in the linear regime only. Simulation results are presented to assess the effect of various system parameters such as amplifier noise figure, number of spans and span length on the spectral efficiency. The results indicate clearly that the spectral efficiency increases almost linearly with both optical signal-to-noise ratio (SNR) and input power, and this effect is more pronounced at high SNR. The UMCD system offers the highest spectral efficiency followed by CIMCD system. The UMDD system shows the lowest efficiency compared with other systems. The efficiency decreases with increasing number of spans, and this effect is more pronounced for the UMCD system.

1. INTRODUCTION

Wavelength division multiplexing (WDM) is a technology that enables the transmission of several optical signals simultaneously at different carrier wavelengths on a single fiber, then the separation of the signals according to their wavelengths at the receiving node. Therefore, the term WDM is also used to refer to the technique where two wavelengths, one in the 1.3 μm optical window and the other in the 1.55 μm window have been multiplexed onto the same link, thereby doubling the link capacity. Recently, the term WDM has referred to the technique where several signals having slightly different wavelengths in the same optical window (1.55 μm) are used as carriers. Furthermore, the term dense wavelength division multiplexing (DWDM) is sometimes used to refer to the latter alternative[1]. The WDM system’s performances can be expressed in terms of spectral efficiency. It is a fundamental measure of the performance of an optical communications system. Therefore, several researchers have been published[2-12] which are focused on study and improvement of spectral efficiency of WDM. The aim of this paper is to investigate theoretically the spectral efficiency of WDM systems for high bit rate. Then performance is evaluated for the effect of various system parameters such as the number of spans, amplifier noise figure, and type of modulation/demodulation scheme on the spectral efficiency.
2. EFFICIENCY IN WDM SYSTEM ANALYSIS OF SPECTRAL

In this section, the spectral efficiency is analyzed and an equation for each WDM system type is derived. Expressions are presented to assess the spectral efficiency when the WDM system is operating in the linear regime. The capacity of a communication channel is the maximum bit rate that can be transmitted without errors, taking into account noise, available bandwidth, and constrained power. In a DWDM system, the spectral efficiency is the capacity per channel divided by the channel spacing:

\[ S = \frac{C}{\Delta f} \]  

(1)

Where \( \Delta f \) is the channel spacing and \( C \) the capacity per channel. \( C \) and \( S \) have units of bits per second (b/s) and b/s/Hz, respectively. The multi-span system shown in figure (1) is considered here. The system comprises \( N_A \) fiber spans, each with gain \( J/G \). After each span, an amplifier of gain \( G \) compensates the span loss. The average transmitted power at the input of each amplifier is \( P_t \) while the average power at the input of each amplifier is \( P_n = P_t/G \). It is assumed that for all detection schemes, amplified spontaneous emission (ASE) noise dominates over other noise sources. At the output of the final amplifier, the ASE in one polarization has a power spectral density (PSD) given by [13]:

\[ S_{eq} = N_A (G-1)n_{sp}h \nu = (G-1)n_{eq}h \nu \]  

(2)

where \( n_{sp} \) is the spontaneous emission noise factor of one amplifier, and \( n_{eq} \) the equivalent noise factor of the multi-span system is defined as \( n_{eq} = N_A n_{sp} \). At the output of the final amplifier, the ASE in one polarization in the channel bandwidth \( B \) has a power as:

\[ P_n = S_{eq} B \]  

(3a)

2.1 Unconstrained Modulation with Coherent Detection

Coherent detection enables information to be encoded in two degrees of freedom: the in-phase and quadrature field components, \( E_I \) and \( E_Q \). Alternatively, one can think of the two degrees of freedom as intensity and phase. The ASE noise is modeled as additive, signal-independent complex circular Gaussian noise. When no constraints are placed on the modulation format, the optimal transmitted electric field is complex circular Gaussian-distributed, as shown in figure (2-a). The capacity is given by the well-known Shannon formula [13]:

\[ C = B \log_2 (1 + SNR) \]  

(4)

and the spectral efficiency is [32] :

\[ S = \frac{B}{\Delta f} \log_2 (1 + SNR) \]  

(5)

Note that at high SNR, the spectral efficiency in equation (5) is given asymptotically by \( S \approx (B/\Delta f) \log_2(SNR) \). and the optical SNR in one polarization in the channel bandwidth \( B \) is:

\[ \text{SNR} = \frac{P_T}{P_n} \]  

(3b)

2.2 Constant-Intensity Modulation with Coherent Detection

Various modulation techniques, such as DPSK and continuous-phase frequency-shift keying (CPFSK), encode information in optical signals having nominally constant intensity. These techniques are often demodulated using differentially coherent detection, which can be implemented by using heterodyne or homodyne detection with delay demodulation, or by using an interferometer with direct detection. The optimal transmitted electric field is uniformly distributed on a circle, as shown in figure (2-b). The capacity of the channel is given by [14]:

\[ C = H (Y) - H (N) \]  

(6)

where \( H(Y) \) is the entropy of the channel output and \( H(N) \) is the entropy of the noise. The entropy of the received signal is given by [14]:

\[ H (Y) = - \int P_Y (Y) \log_2 P_Y (Y) dY \]  

(7)

\[ P_Y (Y) = \int P_{X|Y} (Y |X) q (X) dX \]  

(8)
where $P_{Y|X}(Y|X) = P_N(Y - X)$  \hspace{1cm} (9)

with the constraint of constant-intensity modulation, $X = [x_1, x_2]$ is a complex valued representation of electric field, and assume values a long a circle of radius $A$, as shown in Figure 3-a [15]:

$x_1 = A \cos \theta$ , $x_2 = A \sin \theta$

The optical amplifier noise is additive Gaussian noise. In linear regime $Y = X + N$, as shown in Figure 3-b, where $N$ is a two-dimensional Gaussian variable with variance $\sigma^2_n$.

$$H(N) = - \int P_N(N) \log_2 P_N(N) dN$$

Figure 3: Constant-intensity modulation with additive Gaussian noise (a) input X and (b) output Y [15].

Substituting this into (10)

$$H(N) = - \int \int P_{\rho}(\rho, \rho) \log_2 P_{\rho}(\rho, \rho) \rho d\rho d\theta$$

where $P_{\rho}(\rho, \rho) = \frac{1}{2\pi \sigma^2_e} \frac{\rho}{\rho^2}$

Changing the integral to polar coordinate, and integrate with respect to $\theta$

Substituting (11) and (12) into (6)

$$C = -2\pi \int_0^\infty rf(r) \log_2 f(r) dr - \log_2 (2\pi e \sigma^2_e)$$

At high SNR $I_o(\alpha) \approx e^n / \sqrt{2\pi u}$ [13]
\[ C = 2 \int \left[ -\left( K \log \frac{\sqrt{A} + 1.1 \log_{10}(\nu)}{1.1 \log_{10}(\nu)} \right) + \frac{1}{2} \left( e^{x^2} - 2 \right) + 2 \right] \frac{dx}{\sqrt{\alpha}} \]

\[ s = \frac{1}{2} \log_2 \left( \frac{SNR}{f_{BS}} \right) + 1.1 \]

2.3 Unconstrained Modulation with Direct Detection

Simple direct detection is mathematically equivalent to heterodyne or homodyne detection with noncoherent demodulation. Information may be encoded in the magnitude of the transmitted electric field, using modulation methods such as on-off keying or multilevel pulse amplitude modulation. The transmitted optical signal is modeled as a non-negative, real electric field magnitude. When no constraint is placed on the modulation format, the optimal transmitted field magnitude follows a half-Gaussian distribution, as shown in figure (2-c). The capacity could be calculated by using equations (7), (8) and (9)

\[ P(Y) = \int_0^\infty \frac{1}{\pi \sigma \sigma_s} e^{-\frac{(y-x)^2}{2 \sigma_s^2}} \cdot e^{-\frac{x^2}{2 \sigma^2}} \, dx \]

where \[ q(x) = \frac{\sqrt{2}}{\pi \sigma \sigma_s} e^{-\frac{x^2}{2 \sigma_s^2}} \]

\[ P_N(n) = \frac{1}{\sqrt{2 \pi \sigma \sigma_s}} e^{-\frac{n^2}{2 \sigma_s^2}} \]

\[ P_Y(Y) = \frac{1}{\pi \sigma \sigma_s} \int_0^Y e^{-a (x - \beta)^2} \cdot e^{-\gamma x^2} \, dx \]
Performance Evaluation of Spectral Efficiency in Linear Regime for High Rate WDM System

Where
\[ \alpha = \frac{\sigma^2 \beta^2}{2 \sigma^2 \beta^2}, \quad \beta = \frac{\sigma^2 \gamma}{\sigma^2 \gamma}, \quad \gamma = \frac{1}{2(\sigma^2 \gamma)} \]

Let
\[ Z = \sqrt{\alpha \beta y} \]
\[ dZ = \sqrt{\alpha \beta y} \quad dx \]
\[ x = 0 \Rightarrow Z = -\sqrt{\alpha \beta y} \]
\[ x = \infty \Rightarrow Z = \infty \]

\[ I_1 = -\frac{K \log_2 K}{2} \int y e^{\gamma y} dy \]
\[ I_2 = -\frac{K \log_2 K}{2} \int y e^{\gamma y} dy \]
\[ I_3 = -\frac{K \log_2 K}{2} \int y e^{\gamma y} dy \]
\[ I_4 = -\frac{K \log_2 K}{2} \int y e^{\gamma y} dy \]
\[ I_5 = -\frac{K \log_2 K}{2} \int y e^{\gamma y} dy \]

3. SIMULATION RESULTS

This section presents simulation results related to spectral efficiency of WDM systems using the derived equations (5, 14, and 18) for three types of system (UMCD, CIMCD, and UMDD) respectively. The results are obtained using MATLAB 7.0.

3.1 System Parameters

Figure (4) shows a schematic diagram for the system under investigation. Unless otherwise stated, the parameter’s values used in the simulation are shown in Table 1. The systems are assumed to operate in the 1.55 µm region with 50 GHz channel spacing according to G.692 standards. The 1.55 µm wavelength corresponds to the minimum-loss of the
standard single-mode fiber (≈ 0.2 dB/km). The in-line amplifiers are used to compensate the losses of the fiber spans. For amplifier gain G (dB) and fibre loss α (dB/km), the length of the fiber span is G/α in km.

3.2 Results

Figure (5) shows the variation of spectral efficiency with optical SNR for the three systems. The results are repeated in figure (6) to show the dependency of spectral efficiency with input optical power. As expected, the spectral efficiency increases almost linearly with both optical SNR (dB) and input power (dBm) and this effect is more pronounced at high SNR. The UMCD system offers the highest spectral efficiency followed by CIMCD system.

Table 1. The parameter’s values used in simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical wavelength</td>
<td>λ</td>
<td>1.55</td>
<td>µm</td>
</tr>
<tr>
<td>Bandwidth per channel</td>
<td>B</td>
<td>40</td>
<td>GHz</td>
</tr>
<tr>
<td>Optical input power</td>
<td>P_t</td>
<td>0.1</td>
<td>mW</td>
</tr>
<tr>
<td>Channel spacing</td>
<td>∆f</td>
<td>50</td>
<td>GHz</td>
</tr>
<tr>
<td>Amplifier gain</td>
<td>G</td>
<td>16</td>
<td>dB</td>
</tr>
<tr>
<td>Amplifier noise figure</td>
<td>NF</td>
<td>4.5</td>
<td>dB</td>
</tr>
<tr>
<td>Number of spans</td>
<td>N_A</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Fiber attenuation coefficient</td>
<td>α</td>
<td>0.20</td>
<td>dB/km</td>
</tr>
</tbody>
</table>

The UMDD system shows the lowest efficiency compared with other systems. For example, the UMCD, CIMCD, and UMDD offer spectral efficiencies of 2.77, 2.21 and 0.53 b/s/Hz when SNR = 10 dB (i.e., P_t = -11.54 dBm), respectively. Increasing SNR to 15 dB will enhance the efficiencies of the systems to 4.03, 2.88 and 1.20 b/s/Hz, respectively. These results indicate that the rate of change of efficiency with SNR is the highest for the UMCD system.

Figure (7) depicts the variation of the spectral efficiency of a number of spans N_A. The efficiency decreases with increasing N_A and this effect is more pronounced for the UMCD system. For example, increasing N_A from 10 to 40 will decrease the efficiency from 4.16 to 2.65 b/s/Hz for the UMCD system. These values are to be compared with 2.31 to 1.76 b/s/Hz for the CIMCD system and from 0.63 to 0.08 b/s/Hz for the UMDD system. The calculations are carried further to investigate the effect of amplifier noise figure NF on spectral efficiency, and the results are depicted in Figure 8.

Note that the efficiency decreases almost linearly with NF (in decibels) and this effect is more pronounced for the UMCD system. Increasing the NF from 3 to 6 dB will decrease the efficiency from 3.52 to 2.78 b/s/Hz for the UMCD system, from 2.08 to 1.80 b/s/Hz for CIMCD system and from 0.40 to 0.12 b/s/Hz for the UMDD system.
4. CONCLUSIONS

In this paper, we derived three equations (5,14, and 18) of spectral efficiency for three types of WDM systems (UMCD, (CIMCD), and (UMDD) respectively in the linear regime only. Then we simulated these equations using MATLAB package software to assess the effect of various system parameters on the spectral efficiency. The main conclusions drawn from the simulation results are:

The spectral efficiency increases almost linearly with both optical SNR (dB) and input power (dBm) and this effect is more pronounced at high SNR. The UMCD system offers the highest spectral efficiency followed by CIMCD system. While the UMDD system shows the lowest efficiency compared with other systems. The efficiency decreases with increasing NA and this effect is more pronounced for the UMCD system. The efficiency decreases almost linearly with NF (in decibels) and this effect is more pronounced for the UMCD system.

REFERENCES