RESILIENT CHARACTERISTICS OF SUBGRADE SOILS IN SAUDI ARABIA

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1. INTRODUCTION

The resilient behavior of pavement materials had been the subject of extensive research during the last two decades. The main purpose was to use the resilient characteristics as input for pavement design, since the resilient modulus test simulates the pavement behavior under field service conditions.

Previous research has concentrated on refining the resilient modulus test and reducing its variability, or was directed toward modeling the resilient behavior of pavement materials, especially unbound materials (example: Nazarian and Feliberti, 1993 and Ping, et al., 1998). The relative difficulty with which the resilient modulus test is conducted for unbound pavement materials is one of the reasons behind this research. Another reason was the variable nature of these materials whether it is the base, subbase or subgrade materials. By modeling the resilient modulus of unbound pavement materials, equations can be obtained that relate some or all of the resilient parameters to other properties of the material that can easily be tested and quantified. The purpose of this paper is to establish relationships between the constants of a resilient modulus ($M_R$) constitutive model for subgrade soils and these properties.

2. SCOPE OF RESEARCH

There have been numerous research studies on resilient modulus of subgrade soils. Some of these studies were directed toward modeling $M_R$ as a function of stresses and/or strains or as a function of other soil properties such as CBR optimum moisture content, maximum dry density, etc (Santha, B., 1994). One approach that has received some attention is to relate $M_R$-stress model parameters to such soil properties.

Based on the data available for AASHTO (American Association of State Highway and Transportation Officials) classified A-2-4 type soils (due to the fact that the most dominant soil group in subgrade soils in Saudi Arabia is A-2-4), the following objectives were pursued in this study:

1. Model $M_R$ values as a function of applied stresses.
2. Correlate model parameters with soil properties.

3. BACKGROUND

Since the publication of the AASHTO Guide for Design of Pavement Structure (AASHTO, 1986, 1993) in 1986 and 1993 and the introduction of resilient modulus, $M_R$, to characterize the subgrade...
soil, extensive research has been conducted on the subject. Research has been directed toward developing a reliable \( R_R \) testing system, modeling \( R_R \) as a function of stresses and strains and development of relationships between \( R_R \) or \( R_R \) model constants and soil properties. Several models relating \( R_R \) to stress state have been suggested (Pezo, 1993; Thompson, 1989; Mohammad et al., 1994; and Moassazadeh and Witczak, 1981) for cohesive and cohesionless materials.

A relationship that has gained wide acceptance for fine-grained soil is of the form (Moassazadeh and Witczak, 1981):

\[
R_R = k_1 (\sigma_d)^2
\]

where \( \sigma_d \) = the deviator stress and \( k_1 \) and \( k_2 \) = model constants

This model reflects the dependency of \( R_R \) on deviator stress. In another model, reported by Yoder and Witczak (1975), \( R_R \) values also depend on deviator stress, however, \( R_R \) decreases as deviator stress increases up to a point and then starts to increase. The model is of the following form:

\[
R_R = k_2 + k_3 (\sigma_d - k_1) \quad \text{for} \quad k_1 < \sigma_d
\]

and

\[
R_R = k_2 + k_3 (\sigma_d - k_1) \quad \text{for} \quad k_1 > \sigma_d
\]

where \( k_1 \) = the value of \( R_R \) at the point where \( \sigma_d = k_1 \) and \( k_2 \) and \( k_3 \) are slopes of the portions of the curve representing \( R_R - \sigma_d \) relationship when \( R_R \) is less than and more than \( k_1 \), respectively.

For granular materials, the most significant parameter that influences \( R_R \) is the confining stress. \( R_R \) is usually modeled in terms of either the confining stress (\( \sigma_3 \)) or the bulk stress (\( \theta \)) (Yoder and Witczak, 1975) as follows:

\[
R_R = k_1 (\sigma_3)^2
\]

or

\[
R_R = k_1 (\theta)^2
\]

where \( \theta = \sigma_1 + \sigma_2 \) = \( \sigma_1 + 2\sigma_3 = \sigma_1 + 3\sigma_3 \)

Historically, due to difficulties associated with resilient modulus testing, various approximate empirical formulae have been suggested to estimate \( R_R \) from CBR by many researchers (Heukelom and Klomp, 1962; Heukelom and Klomp, 1981; Green and Hall, 1975; Powell et al., 1984; Witczak et al., 1995). However, these types of models have not been universally accepted as on one hand, they were developed for certain localities and cannot be generalized, and on the other hand they do not recognize the stress dependence of \( R_R \) (Rada and Witczak, 1981; Drumm et al., 1990).

Pezo (1993) proposed a stress dependent model that incorporates both deviatoric and confining stresses.

\[
R_R = k_1 (\sigma_d)^2
\]

\[
R_R = k_2 (\sigma_3)^2
\]

\[
k_1 = k_1 (\sigma_d)^2
\]

\[
k_2 = k_2 (\sigma_3)^2
\]

The model can be used for both cohesive and cohesionless soils. The soil type is reflected by the weight given to either deviator or confining stresses in the model which is developed by regression analysis. The model is of the following form:

\[
R_R = K_1(\sigma_d)K_2 (\sigma_3)^K_3
\]

where \( K_1 \), \( K_2 \) and \( K_3 \) are regression constants.

This type of model was found to fit \( R_R \) data very well as demonstrated by an extensive study that included almost all types of soils (Al-Suhaibani et al., 1997).

\section*{4. DATA COLLECTION AND TESTING}

This study represents part of an overall research project sponsored by King Abdulaziz City for Science and Technology (KACST) to study the subgrade soil in Saudi Arabia (Al-Suhaibani, et al., 1997). The soil samples were collected from all over the Kingdom along and adjacent to most of the major highways (approximately 6440 km). The samples were collected at constant intervals (40 km) from pits excavated at about 20m from the pavement edge. The locations of the collected samples are shown in Figure 1. Soil samples were distributed among most AASHTO soil classes. However, about 54% of the samples were A-2-4 type soils. For this reason A-2-4 soil samples were selected to be the subject of this analysis. The total number of samples used is 74.

Prior to \( R_R \) testing, all collected soil samples were tested using routine geotechnical tests in accordance with relevant AASHTO standards to determine their engineering properties. Properties that were

![Map of Saudi Arabia Showing Sample Locations (dots along major roads).](image-url)
determined include grain size distribution, specific gravity, optimum moisture content, maximum dry density, and Atterberg limits. In addition, CBR and unconfined compressive strength were determined. Table 1 shows the tested properties and methods of testing.

**Table 1. Tested Soil Properties and Test Method**

<table>
<thead>
<tr>
<th>Soil Property</th>
<th>Test Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Moisture Content</td>
<td>AASHTO T265</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>AASHTO T85 and T100</td>
</tr>
<tr>
<td>Laboratory Density (Max Dry Density)</td>
<td>AASHTO T99 Method D</td>
</tr>
<tr>
<td>Optimum Moisture Content</td>
<td>AASHTO T99 Method D</td>
</tr>
<tr>
<td>CBR</td>
<td>AASHTO T193</td>
</tr>
<tr>
<td>Unconfined Compressive Strength, C</td>
<td>ASTM D 2166-91</td>
</tr>
<tr>
<td>Atterberg Limits</td>
<td>AASHTO T89 and T90</td>
</tr>
<tr>
<td>Sieve Analysis</td>
<td>AASHTO T88-86</td>
</tr>
<tr>
<td>Soil Classification According to AASHTO, Unified and SHRP</td>
<td>AASHTO, Unified and SHRP</td>
</tr>
<tr>
<td>Resilient Modulus, MR</td>
<td>AASHTO T294-921</td>
</tr>
</tbody>
</table>

**4.1 MR TESTING**

Soil samples were first separated into SHRP (Strategic Highway Research Program) Type I and Type II according to the criteria given in SHRP protocol P46 (designated later as AASHTO 294-92I). The method of testing was different for Type I and Type II soils. The stress state and stress sequence is different for each type of soil as shown in Table 2.

Sample preparation was done according to the procedure given in AASHTO 294-92I. Samples were compacted statically in five equal layers up to the maximum dry density at the optimum moisture content using a mold 200 mm in height and 100 mm in diameter.

The sample was then transferred to the triaxial cell in the testing apparatus. Figure 2 shows a schematic diagram of MR apparatus. The vertical resilient strain was measured by two LVDT’s (Linear Variable Deferential Transducers) which were externally mounted on the apparatus for easy adjustment. The deviatoric stress was measured by a load cell mounted directly on the top of the soil sample. Both LVDT’s and load cell were connected to a data acquisition system. The results, which include deviatoric and confining stresses and the calculated MR values, were printed out by the computer that was connected to the system. Typical MR results are shown in Figures 3 and 4 for Type I and Type II soils, respectively.

**Table 2. Testing Sequence of the MR Test (AASHTO T294-92I)**

<table>
<thead>
<tr>
<th>Sequence No</th>
<th>Number of Load Applications</th>
<th>Confining Pressure, $\sigma_3$, psi</th>
<th>Deviator Stress, $\sigma_d$, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1000</td>
<td>15 Material Type I</td>
<td>15 Material Type I</td>
</tr>
<tr>
<td>0</td>
<td>(Preconditioning)</td>
<td>15 Material Type II</td>
<td>15 Material Type II</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>3 6 Material Type I</td>
<td>3 Material Type II</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>3 6 Material Type II</td>
<td>3 Material Type II</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>3 6 Material Type II</td>
<td>3 Material Type II</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>5 6 Material Type II</td>
<td>5 Material Type II</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>5 6 Material Type II</td>
<td>5 Material Type II</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>5 3 Material Type II</td>
<td>5 Material Type II</td>
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<tr>
<td>7</td>
<td>100</td>
<td>10 3 Material Type II</td>
<td>10 Material Type II</td>
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<tr>
<td>8</td>
<td>100</td>
<td>10 3 Material Type II</td>
<td>10 Material Type II</td>
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<tr>
<td>9</td>
<td>100</td>
<td>10 3 Material Type II</td>
<td>10 Material Type II</td>
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<tr>
<td>10</td>
<td>100</td>
<td>10 3 Material Type II</td>
<td>10 Material Type II</td>
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<tr>
<td>11</td>
<td>100</td>
<td>15 0 Material Type I</td>
<td>15 Material Type I</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>15 0 Material Type II</td>
<td>15 Material Type I</td>
</tr>
<tr>
<td>13</td>
<td>100</td>
<td>20 0 Material Type I</td>
<td>20 Material Type I</td>
</tr>
<tr>
<td>14</td>
<td>100</td>
<td>20 0 Material Type II</td>
<td>20 Material Type I</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
<td>20 0 Material Type II</td>
<td>20 Material Type II</td>
</tr>
</tbody>
</table>
5. RESULTS AND DISCUSSION

5.1 SOIL PROPERTIES

Soil properties including CBR, unconfined compressive strength, Atterberg limits, specific gravity, optimum moisture content, maximum dry density, and field moisture content were obtained. Samples for CBR, unconfined compressive strength and MR tests were prepared at optimum moisture content and maximum dry density. Void ratio, degree of saturation, group index, swelling index and liquidity index were calculated from other soil properties. Table 3 shows the correlation matrix (correlation coefficients, R’s) for these properties. With the exception of those properties that are naturally related to each other or were developed from each other, some correlation coefficients are noteworthy. Of importance are the correlation coefficients relating MR model (Equation (6)) constants, $K_1$, $K_2$, and $K_3$, to soil properties. By examining the correlation matrix, it could be observed that:

1) The correlation data indicate that $K_2$ is more significantly related to the soil properties compared with $K_1$ and $K_3$.
2) $K_1$ increases as square of plasticity index, plasticity index, percentage of clay, unconfined compressive strength and percent passing sieve #10 increase.
3) $K_2$ increases as sand content and maximum dry density increase and as natural moisture content, material passing sieve # 200 (silt and clay contents), liquid and plastic limits, plasticity index, and optimum moisture content decrease.
4) $K_3$ increases as the natural moisture content increases and as the clay content decreases. It also decreases as material passing sieve #40 and sieve #10 increase.

5) $K_1$ is negatively correlated with $K_2$ and $K_3$.
6) The highest correlation among soil properties is between the unconfined compressive strength ($q$) and the liquid limit (LL) ($R= 0.59$).
7) Relatively high correlation also exist among each of material passing sieves #4, #10, #40, #200, sand, silt, and clay and Atterberg Limits.

5.2 MR MODEL

The MR data was modeled by regression analysis where the constants $K_1$, $K_2$ and $K_3$ were determined. Table 4 shows sample of these constants together with the coefficient of determination, $R^2$, and standard error of estimate, $S$. The value of $R^2$ shows how well the model with the resulting constants fits the original MR data, while $S$ is an estimate of the standard deviation of the dependent variable for cases which have the same values of the independent variable. Almost all $R^2$ values are 0.9 or above indicating that the model fits the data very well. The value of $K_1$ represents the value of MR when the values of $s_d$ and $s_3$ are unity. The values of $K_2$ and $K_3$ represent the relative importance or contribution of $s_d$ and $s_3$, respectively, to MR values.

The most important aspect shown in Table 4 is that, there is no consistent deviator stress behavior of A-2-4 soil, where 52% of the samples exhibited deviator stress softening (negative $K_2$) and 48% exhibited deviator stress hardening (positive $K_2$), that is, increase in MR as deviator stress increases.

In order to determine typical values of $K_1$, $K_2$, and $K_3$, their values were summarized in Table 5. It can be seen that the majority of $K_1$ values fall between 13 and 65, $K_3$ ranges from $-0.15$ to $0.09$ and $K_3$ between $0.14$ and $0.48$. 

Figure 3. Typical MR Results for Type I Soil

Figure 4. Typical MR Results for Type II Soil
5.3 RELATING M<sub>R</sub> MODEL CONSTANTS TO SOIL PROPERTIES

In this study, the M<sub>R</sub> model constants were related to soil properties. Stepwise regression was used to select soil properties that best explain the variation in the model constants; K<sub>1</sub>, K<sub>2</sub> and K<sub>3</sub>. Both aggregate (using all data points) and disaggregate (soils were separated into plastic and non-plastic) models were constructed.

Table 3. Correlation Matrix of Soil Properties

<table>
<thead>
<tr>
<th></th>
<th>NMC</th>
<th>SILT</th>
<th>SV4</th>
<th>SV10</th>
<th>SV40</th>
<th>SV200</th>
<th>SAND</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>-1.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PL</td>
<td>0.23</td>
<td>0.95</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MDN</td>
<td>0.15 -0.10 -0.07 1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OMC</td>
<td>0.02 0.39 0.37 -0.85 1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CBR</td>
<td>-0.13 -0.16 -0.11 -0.08 -0.06 1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VRAT</td>
<td>-0.16 -0.16 -0.12 -0.93 0.86 0.08 0.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DST</td>
<td>0.41 0.48 0.51 0.15 0.32 -0.36 0.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PRSTY</td>
<td>0.16 0.17 0.12 0.93 0.85 0.08 0.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GI</td>
<td>0.37</td>
<td>0.93</td>
<td>0.04 0.36 -0.19 0.57</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SWIND</td>
<td>1.00 0.95 0.09 0.39 -0.16 0.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LL/CL</td>
<td>-0.45 0.46 0.51 -0.17 0.25 0.05 0.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>0.31 0.74 0.50 -0.15 0.30 -0.19 0.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LI</td>
<td>0.19 -0.03 -0.12 -0.21 0.11 0.17 0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K&lt;sub&gt;1&lt;/sub&gt;</td>
<td>0.22</td>
<td>0.00</td>
<td>0.11 0.16 0.16 0.06 0.20</td>
<td>0.00 0.11 0.04 0.58</td>
<td>0.70</td>
<td>0.00</td>
<td>0.11 0.16 0.16 0.06 0.20</td>
</tr>
<tr>
<td>K&lt;sub&gt;2&lt;/sub&gt;</td>
<td>-0.33</td>
<td>-0.50</td>
<td>-0.42 0.28 -0.44 -0.04 -0.58</td>
<td>-0.22</td>
<td>0.06 0.09 0.13</td>
<td>0.12</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 4. Sample of M<sub>R</sub> Model Constants

<table>
<thead>
<tr>
<th>Sample</th>
<th>SHRP Type</th>
<th>K&lt;sub&gt;1&lt;/sub&gt;</th>
<th>K&lt;sub&gt;2&lt;/sub&gt;</th>
<th>K&lt;sub&gt;3&lt;/sub&gt;</th>
<th>R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-04</td>
<td>II</td>
<td>32.342</td>
<td>-0.123</td>
<td>0.359</td>
<td>0.919</td>
<td>3.405</td>
</tr>
<tr>
<td>A-07</td>
<td>II</td>
<td>23.419</td>
<td>0.321</td>
<td>0.259</td>
<td>0.960</td>
<td>2.783</td>
</tr>
<tr>
<td>A-10</td>
<td>II</td>
<td>107.678</td>
<td>-0.098</td>
<td>0.262</td>
<td>0.924</td>
<td>6.686</td>
</tr>
<tr>
<td>A-12</td>
<td>II</td>
<td>75.640</td>
<td>-0.046</td>
<td>0.339</td>
<td>0.933</td>
<td>7.605</td>
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<tr>
<td>A-14</td>
<td>II</td>
<td>16.217</td>
<td>0.027</td>
<td>0.487</td>
<td>0.997</td>
<td>8.322</td>
</tr>
<tr>
<td>A-17</td>
<td>II</td>
<td>60.394</td>
<td>-0.106</td>
<td>0.419</td>
<td>0.923</td>
<td>8.492</td>
</tr>
<tr>
<td>A-18</td>
<td>II</td>
<td>31.191</td>
<td>0.026</td>
<td>0.292</td>
<td>0.860</td>
<td>4.492</td>
</tr>
<tr>
<td>A-19</td>
<td>II</td>
<td>34.973</td>
<td>0.044</td>
<td>0.257</td>
<td>0.988</td>
<td>1.151</td>
</tr>
<tr>
<td>B-02</td>
<td>II</td>
<td>31.466</td>
<td>-0.006</td>
<td>0.413</td>
<td>0.805</td>
<td>10.480</td>
</tr>
<tr>
<td>B-04</td>
<td>II</td>
<td>32.338</td>
<td>0.085</td>
<td>0.251</td>
<td>0.897</td>
<td>4.246</td>
</tr>
<tr>
<td>B-05</td>
<td>II</td>
<td>43.956</td>
<td>-0.220</td>
<td>0.277</td>
<td>0.958</td>
<td>1.988</td>
</tr>
<tr>
<td>B-07</td>
<td>II</td>
<td>44.107</td>
<td>-0.048</td>
<td>0.131</td>
<td>0.832</td>
<td>5.186</td>
</tr>
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</table>

Table 5. Summary of K1, K2 and K3 Statistics

<table>
<thead>
<tr>
<th>Property</th>
<th>K1</th>
<th>K2</th>
<th>K3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>35.519</td>
<td>-0.015</td>
<td>0.296</td>
</tr>
<tr>
<td>Median</td>
<td>32.093</td>
<td>-0.010</td>
<td>0.291</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>18.637</td>
<td>0.090</td>
<td>0.104</td>
</tr>
<tr>
<td>Maximum</td>
<td>107.678</td>
<td>0.321</td>
<td>0.504</td>
</tr>
<tr>
<td>Minimum</td>
<td>8.139</td>
<td>-0.220</td>
<td>0.039</td>
</tr>
<tr>
<td>Range (90%)</td>
<td>13–65</td>
<td>-0.15–0.09</td>
<td>0.14–</td>
</tr>
</tbody>
</table>

5.3.1 Regression Equations for K<sub>3</sub>:

At the beginning, an attempt was made to find a relationship between K<sub>3</sub> and soil properties for all data points. The results, after removing outliers, were as follows:

1) All data points:

K<sub>3</sub> = -77.17 + 4.31 (PI) + 0.573 (SV10) + 31.6

(MDN) - 1.21 (LL/CL) (7)

where

PI = plasticity index;
SV10 = percent passing sieve # 10;
MDN = max dry density;
LL = liquid limit and
CL = percentage of clay.

The t-statistics for the constant and the independent variables were -2.758, 7.598, 4.451, 2.696, and -2.452, respectively. The coefficient of determination for this equation, R<sup>2</sup>, is 50.71%, standard error of estimate, S, is 10.221, and the number of data points used for building the model, n, is 68. The F-statistics for the model for 4 and 63 degrees of freedom, F (4,63), is 18.23. The F value should be more than the critical value at the 95% significant level, which is the case here. The Variance Inflation Factor (VIF), which is an indicator of collinearity among various independent variables, is less than 1.2. A value of VIF less than 4 indicates no collinearity, a value more than 4 indicates possible collinearity, and a value

NMC = natural moisture content, SV4 = % passing sieve #4, MDN = max. dry density, q = unconfined compressive strength, VRAT = void ratio, DST = degree of saturation, PRSTY = porosity, GI = group index, SWIND = swelling index (LL*Gs), GS = specific gravity, LI = liquidity index (NMC-PL)/PI, CL = % of clay.
above 10 indicates serious collinearity. It is clear that VIF values for all independent variables were well below 4, which indicate almost no correlation among these variables.

It was thought that if the data were subdivided into more homogenous groups more representative regression equations could be developed. Therefore, separate regression equations relating $K_1$ to various soil properties were developed for soil samples classified as non-plastic (57 samples) and as plastic (17 samples).

2) For non-plastic soil (PI = 0) the developed equation was as follows:

$$K_1 = 19.23 - 2.517 \times \text{(NMC)} + 0.182 \times \text{(q)} - 0.891 \times \text{(Silt)} + 0.301 \times \text{(SV40)} + 0.488 \times \text{(LL)}$$  \hspace{1cm} (8)

where NMC = natural moisture content; q = unconfined compressive strength, kPa; SV40 = percent passing sieve #40; LL = liquid limit; Silt = percentage of silt.

The t-statistics for the constant and the independent variables were 2.87, -6.1, 4.96, -4.22, 3.82, and 2.00, respectively. $R^2 = 57.6\%$, $S = 6.241$, $n = 46$, $F(5,40) = 13.204$, and VIF $\leq 1.4$. From the value of $R^2$ for this equation, the independent variables explain about 57.6% of the variability in $K_1$ values.

3) For plastic soil (PI > 0) the stepwise regression resulted in an equation of the following form:

$$K_1 = -106.47 + 2.64 \times \text{(SV10)} + 0.688 \times \text{(PI)} - 1.46 \times \text{(SV40)} + 0.339 \times \text{(CBR)}$$  \hspace{1cm} (9)

where all variables were defined earlier.

The t-statistics for the constant and the independent variables were -8.76, -6.14, 9.46, -4.22, 3.82, and 2.00, respectively. $R^2 = 57.6\%$, $S = 6.241$, $n = 46$, $F(4,40) = 13.204$, and VIF $\leq 1.4$. From the value of $R^2$ for this equation, the independent variables explain about 57.6% of the variability in $K_1$ values.

5.3.2 Regression Models for $K_2$:

Regression analysis was also conducted for $K_2$ and for the same categories mentioned above for $K_1$.

1) For all data points:

$$K_2 = -0.4998 - 0.00123 \times \text{(q)} + 0.2797 \times \text{(MDN)} - 0.004387 \times \text{(CL)}$$  \hspace{1cm} (10)

where all variables were defined earlier

$R^2 = 63.4\%$; $VIF \leq 1.16$; $F(3,67) = 41.416$; $S = 0.045$ and $n = 71$.

The t-statistics for the constant and the independent variables were -5.04, -7.39, and -2.26, respectively.

2) Non-plastic soil (PI = 0):

$$K_2 = -0.064 - 0.00107 \times \text{(q)} + 0.125 \times \text{(MDN)} - 0.00183 \times \text{(SAND)} + 0.00555 \times \text{(LL/CL)}$$  \hspace{1cm} (11)

where Sand = percentage of sand.

$R^2 = 68.9\%$; $VIF \leq 1.4$; $F(4,41) = 25.884$; $S = 0.024$ and $n = 46$.

The t-statistics for the constant and the independent variables were -0.82, -7.47, 3.74, -4.28, and -2.19, respectively.

3) Plastic soil (PI > 0):

$$K_2 = 0.12429 - 0.0238 \times \text{(LI)} - 0.0028 \times \text{(q)} - 0.0044 \times \text{(SILT)}$$  \hspace{1cm} (12)

where LI = liquidity index = (NMC-PL)/(LL-PL); SILT = percentage of silt.

$R^2 = 86.47\%$; $VIF \leq 1.1$; $F(3,11) = 30.824$; $S = 0.03546$ and $n = 15$.

The t-statistics for the constant and the independent variables were 2.23, -7.24, 5.16, and -2.09, respectively.

Comparing equations 10, 11 and 12, it is quite clear that disaggregate models (based on separating soil samples into plastic and non-plastic) are the most reliable ones. The unconfined compressive strength is an important variable for both plastic and non-plastic soils models.

5.3.3 Regression equations for $K_3$:

The final set of regression equations is for $K_3$, which is the power of the confining stress in the resilient modulus equation (Equation (6)).

1) Regression equation for all data points is:

$$K_3 = 0.667 + 0.0208 \times \text{(NMC)} - 0.00368 \times \text{(SV40)} - 0.347 \times \text{(DSAT)} + 0.00402 \times \text{(SV200)} - 0.00126 \times \text{(PI)}$$  \hspace{1cm} (13)

where DSAT = degree of saturation and SV200 = percent passing sieve #200.

$R^2 = 59.2\%$; $VIF \leq 1.6$; $F(5,56) = 18.688$; $S = 0.0558$ and $n = 62$. 

The t-statistics for the constant and the independent variables were 6.98, 7.24, -5.25, -3.51, 2.94, and – 2.02, respectively.

2) The model for non-plastic soils (PI = 0) was found to be:

\[ K_3 = 0.742 + 0.0239 \times (NMC) + 0.000805 \times (q) - 0.00378 \times (SV40) - 0.508 \times (DST) + 0.00557 \times (SV200) \]  

(14)

where all variables were defined earlier.

\[ R^2 = 75.7\%; \quad VIF \leq 1.7; \quad F (5,42) = 30.247; \quad S = 0.04274 \text{ and } n = 48. \]

The t-statistics for the constant and the independent variables were 9.15, 8.65, 3.12, -5.99, -5.91, and 3.88, respectively.

3) For plastic soils (PI > 0) the regression equation is:

\[ K_3 = 1.89 - 0.0234 \times (SV200) - 0.00854 \times (SWIND) - 0.0155 \times (NMC) - 0.00514 \times (SAND) + 0.00959 \times (LI) \]  

(15)

where SWIND = swelling index = LL*Gs and Gs = specific gravity.

\[ R^2 = 79.86\%; \quad VIF \leq 3.6; \quad F (5,9) = 12.106; \quad S = 0.045 \text{ and } n = 15. \]

The t-statistics for the constant and the independent variables were 7.66, -6.52, -5.78, -3.46, -3.15, and 2.60, respectively.

As with \( K_1 \) and \( K_2 \), separating the soil into plastic and non-plastic greatly improved the reliability of the model. However, final conclusion should be based upon a comparison of measured and predicted M_R values.

Table 6 presents a summary of the above parameters for each regression equation. Using the regression equations for each of \( K_1 \), \( K_2 \), and \( K_3 \) for all data points (equations 7, 10, and 13, respectively), a prediction for M_R was obtained for each soil type. Comparison of predicted and measured M_R values are shown in Figures 5, 6 and 7 for aggregate data, non-plastic soil, and plastic soil, respectively. Figure 5 shows wide scatter of points around the equality line while disaggregate data, Figures 6 and 7, show less scatter. However, the small number of points for plastic soil makes the prediction models for this type of soil less reliable. It has been extensively reported in the literature that the repeatability of the M_R test is relatively poor. Bearing this in mind, the benefits of these models should be viewed within this context. It is, also, worth mentioning here that since the predicted M_R values were obtained using three regression models for \( K_1 \), \( K_2 \), and \( K_3 \), and the variation around the equality line is the result of the variation of the three models.

From the scatter plots and the coefficients of determinations for \( K_3 \)’ regression equations, it was clear that the disaggregate models are more reliable than the aggregate ones. In order to verify this concept statistically, a t-test for equal means of measured and predicted M_R for each set was run. The set of data that gives the most significant result is to be recommended. The results are shown in Table 7. Based on “t” and probability values at a significance level of 95%, it can be seen that disaggregate sets are more significant than the aggregate one (disaggregate data show higher probability). Therefore, it is concluded that disaggregate models are preferred over the aggregate ones. This conclusion agrees with that reported above by looking into the scatter plot of measured vs. predicted M_R values and the values of coefficient of determinations for the regression equations.

Given the difficulties in M_R testing, the prediction of M_R values has long been the concern of many researchers. This paper is an attempt toward this direction. However, as seen from the results, the prediction reliability is very much affected by the nature of both the tested material and the test itself (M_R test). Natural soil possesses an inherent variability and little can be done to reduce its effect. The high variability of M_R testing is well known even with the most reliable testing system. Given these facts, it can be concluded that the models constructed in this paper, especially disaggregate models, are relatively good and can be used for M_R prediction after verification.
Table 7. Results of Test for Equal Means of Measured and Predicted $M_R$ Values for The Suggested Regression Equations

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>All Data</th>
<th>Plastic</th>
<th>Non-Plastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_R$ Data</td>
<td>Measured</td>
<td>Predicted</td>
<td>Measured</td>
</tr>
<tr>
<td>No. of Samples</td>
<td>56</td>
<td>56</td>
<td>13</td>
</tr>
<tr>
<td>Mean</td>
<td>74.76</td>
<td>77.48</td>
<td>71.92</td>
</tr>
<tr>
<td>Variance</td>
<td>594</td>
<td>514</td>
<td>1156</td>
</tr>
<tr>
<td>t stat.</td>
<td>-0.61</td>
<td>0.088</td>
<td>0.034</td>
</tr>
<tr>
<td>t critical (2 tail)*</td>
<td>1.982</td>
<td>2.064</td>
<td>1.997</td>
</tr>
<tr>
<td>P(T&lt;=t)*</td>
<td>0.542</td>
<td>0.930</td>
<td>0.973</td>
</tr>
</tbody>
</table>

* $\alpha = 0.05$

6. CONCLUSIONS

The following conclusions were based on the results of this study:

1- A constitutive regression equation that includes both deviator and confining stresses was found to reflect $M_R$ dependency on those two stresses with very high coefficient of determination.

2- Correlation data indicate that $K_2$ is more significantly related to soil properties compared with $K_1$ and $K_3$.

3- Correlation data indicate that $K_1$ increases as square of plasticity index, plasticity index, percentage of clay, unconfined compressive strength and percent passing sieve #10 increase.

4- $K_2$ increases as sand content and maximum dry density increase and as natural moisture content, material passing sieve # 200 (silt and clay contents), liquid and plastic limits, plasticity index, and optimum moisture content decrease.

5- $K_3$ increases as the natural moisture content increases and as the clay content decreases. It also decreases as material passing sieve #40 and sieve #10 increase.

6- $K_1$ is negatively correlated with $K_2$ and $K_3$.

7- Relatively good fit regression equations, which relate $K_1$, $K_2$ and $K_3$ to soil properties for both aggregate and disaggregate data for plastic and non-plastic soils, were constructed.

8- Disaggregate models were found to better describe $M_R$ variation with soil properties, thus they are recommended over aggregate models.

9- Although the developed models could be used to estimate $M_R$ value employing simple soil properties that are easily determined, accurate $M_R$ can only be obtained by conducting $M_R$ test.
7. ACKNOWLEDGMENT

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8. REFERENCES


